Numbers In A Nutshell

Back to Basics

An Educational Revolution

Aart Bark

FROM THE PAST
TO THE PRESENT
FOR THE FUTURE

Authors background

Born in 1928

1. **Dutch Diplomas:**
   HSC, Naval College, 3rd & 2nd mate Dutch Merchant Navy, Commercial correspondence Spanish, French, English.
2. **Languages:** Dutch, English, German, French, Spanish, Portuguese, Italian.
3. **High School teacher:** Holland (2 years)
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4. **Subjects:** English, French, Latin, Spanish, Creative writing,
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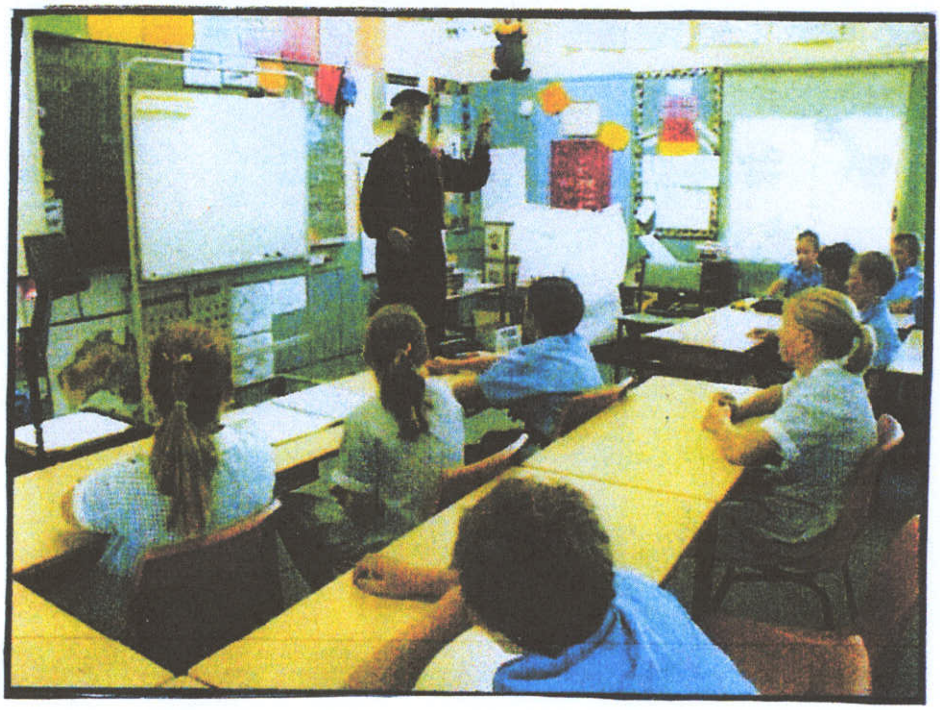
5. Private tutor since 1976.
6. Financial advisor: Insurance & Real Estate.
7. Owner-builder: Rammed Earth.
8. Author of text books.
9. Musician, Artist Painter, Soccer Coach.



Numbers in a Nutshell

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Endangered Minds

In designing Numbers in a Nutshell, the author has taken the following criteria into account:

- The excessive emphasis on entertainment and the fact that the human mind switches off when bombarded by too much information, endless explanations and the ever increasing stimuli of modern advertising.

- The necessity to think is now often being replaced by reacting to flashing warning lights en beeps. Homo Sapiens is gradually becoming a manipulated robot.

- The New Neandertal Child.
  Many children now have a T.V. brain, one capable of watching the screen but not much else (Healey).

- Frequent use of walkmen and mobile phones diminishes hearing and scholastic performance. (Uma História da Linguagem).

- It takes at least 10 years for the human brain to reach maturity; at birth, its weight is only 25% of the adult one. Although it's then only 2% of the average body weight, it consumes, 20%-25% of the daily energy intake! (Uma História da Linguagem).

- Consequently, asking children what they want- a modern phenomenon in an affluent society-is extremely unwise, uneducational and fraught with problems sooner or later. The effects of eating junkfood becomes also clear: obesity and reduced brain function!

<u>The Remedy</u>

- Whereas modern, commercial methods solely depend on **unreliable intelligence and quantity**, <u>New Millennium Maths</u> **makes use of eyes and quality**. However, watching and looking are not the same as seeing!

- The author shows students what to do and how to do it in the shortest professional way.

- By adopting the disciplines necessary to cause the human brain to function to full capacity-creating Interest, using Association. Visualisation, Verbal Rehearsal (saying and doing), Imagination, Routine and Repetition – seeing can be improved to an amazing degree! Learning more will be the natural result. (Professional Memory Training)

- **No more tables**! If someone says it can't be done, somebody else has already done it! (Chinese proverb).

<u>Maximum Impact, Minimum Effort</u>

- Numbers in a Nutshell is based on number patterns nobody has ever discovered before! Diagrams in colour avoid lengthy and complicated explanations.
  If you explain too much, you explain nothing!

- Answers to pairs and whole groups can now be found by using one 'recipe' or general rule (Necessity is the Mother of Invention!)

- With this method, learning numerical facts may be compared to learning the names of classmates; it has nothing to do with intelligence.

- However, before that is possible, one has to recognise them first!

- Consequently, for each of the four operations, clear instructions are given for the special recognition exercises that have to be done before memorising the recipes and general rules.

- These exercises consist of putting labelled cards on a number board. They have to be done until students can do them from memory. The cards may be stored in a matchbox.

A Practical Approach

- With CDs, lessons can commence immediately.

- The DVD produced in Cattai Public School shows clearly that teaching a whole class is better, more effective and less time consuming than teaching small groups of different ability levels!

- Since the author only uses prototypes, the number of periods may be reduced, thus solving the teacher shortage.

- The classroom becomes a workshop after the teacher has demonstrated a new topic and tested a few volunteers. (At most 15 minutes; the example stays on the screen).

- Then students invent their own variations on white board: write & wipe! This novel approach creates not only initiative (the opposite a chore) but automatically allows students to work at their own level as well.

- The teacher helps. Marks are not given; all work has to be corrected if necessary. There is no homework!

Multicultural Numeracy

- English is now the most widely language used in the world.

- It is a well-known fact that one should learn another language at an early age.

- In this course, the English used is absorbed subconsciously.

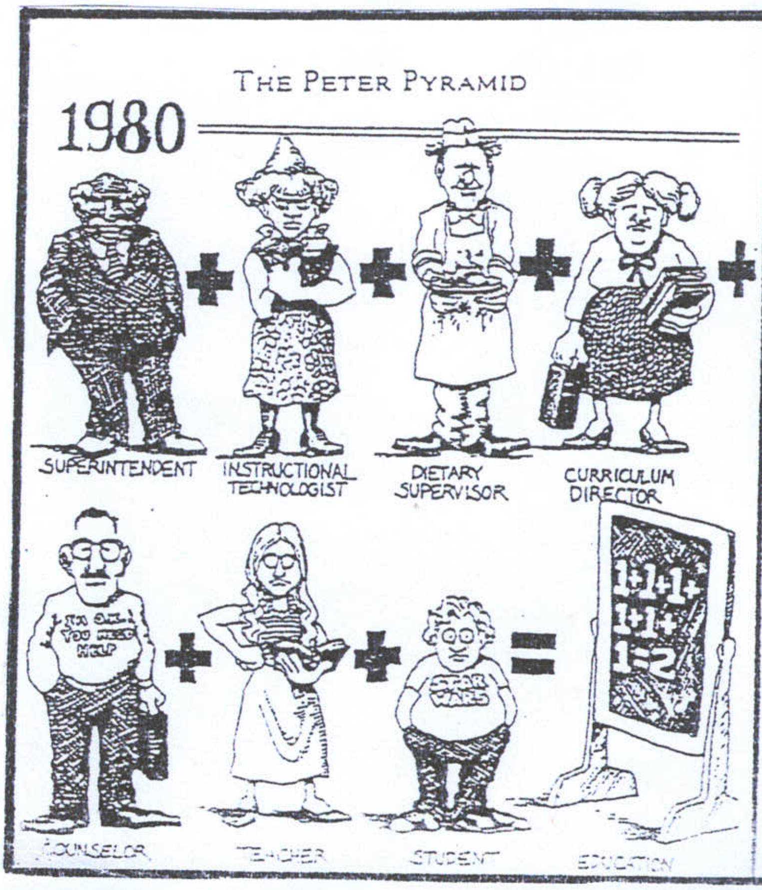
- The so-called recipes and general rules must be recited by the whole class (chorus reading).

- Translations are not necessary; the examples speak for themselves.

- **Numbers in a Nutshell** is exactly what it says: it's a **Number Book**.

- There are no commercials in which children are encouraged to buy lollies, ice cream, balloons for their birthday, electric guitars with amplifiers and speakers (subliminal advertising!)

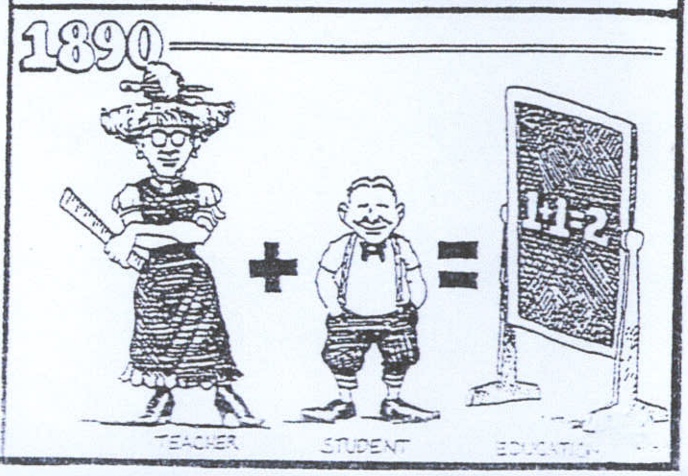
- Playing shops is of yesteryear and time-wasting.

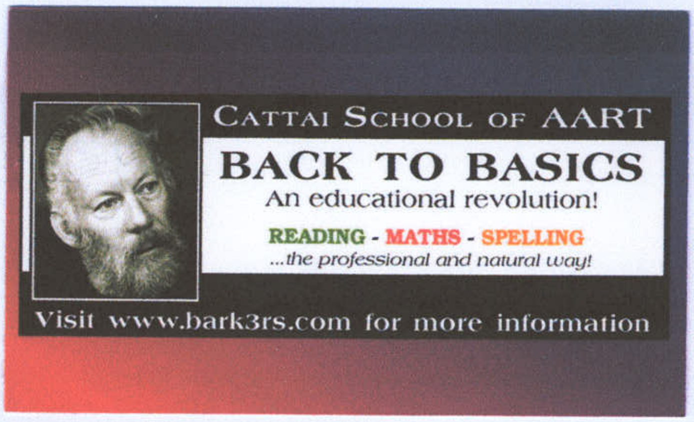


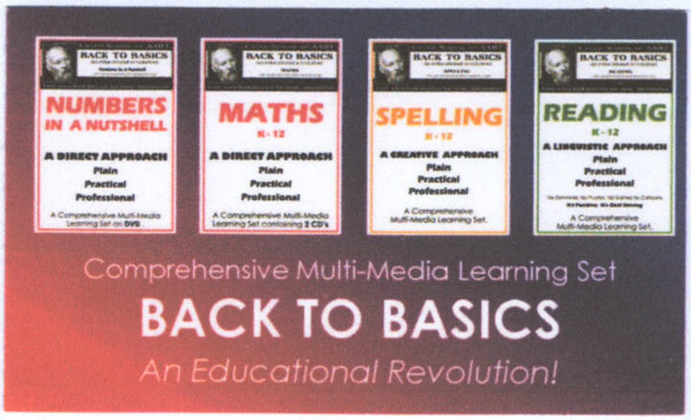
The Myer Report (1980)

Of even greater long-term concern, the report argued, was the large proportion (more than 70 per cent) of primary schoolteachers who took no mathematics in their last years of high school. The report was concerned that teachers lacking familiarity with mathematics may educate their students towards a similar unfamiliarity.

Back to basics







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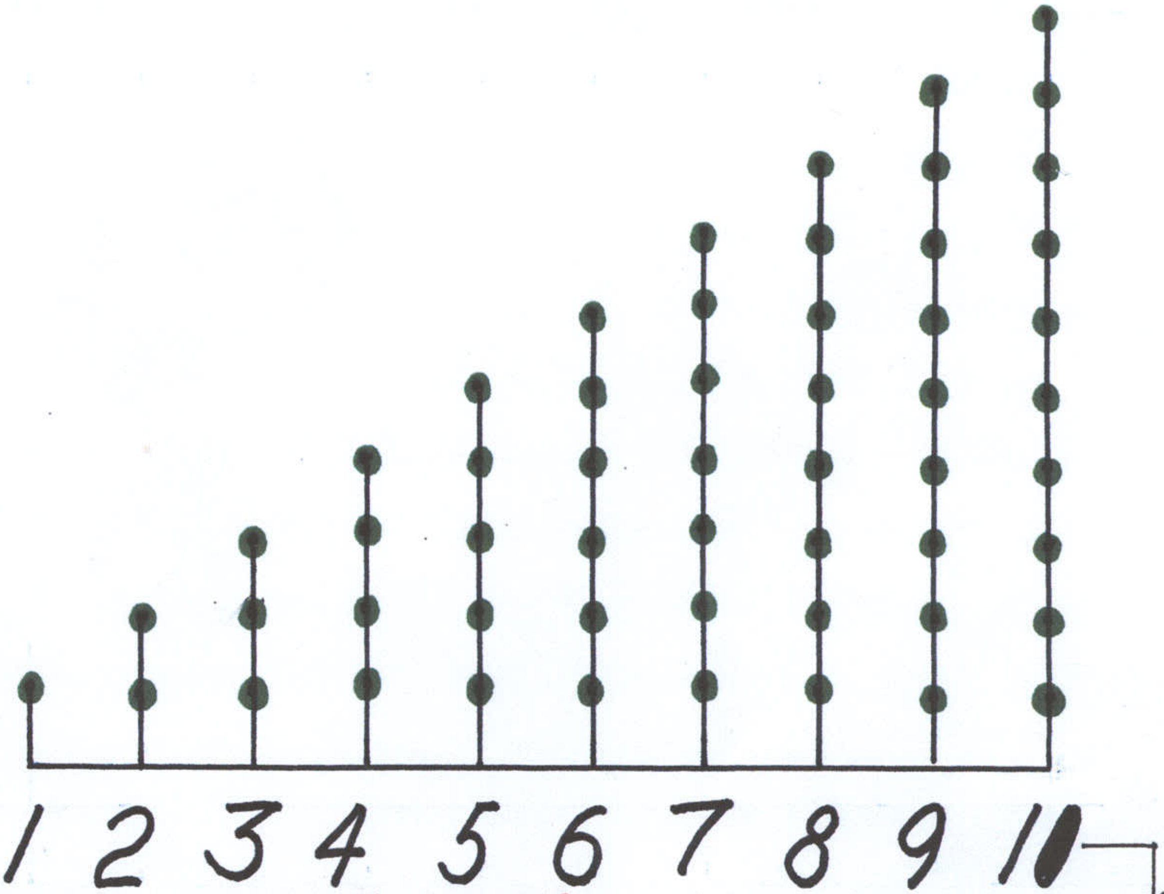
COUNTING NUMBERS



POINT & COUNT ALOUD

TRADE MARK

MOST CHILDREN WRITE NUMBERS 'BACK TO FRONT' BECAUSE THEY START TOO EARLY; THE NECESSARY BRAINCELLS FOR THIS COMPLEX ACTIVITY WERE NOT YET IN PLACE. THAT'S WHY SOME HOLD THEIR PEN AS IF IT WERE A WEAPON! AS ADULTS, THEY STILL DO.

∴ SYMBOL MEANING THEREFORE

TRACING BEFORE WRITING

USING – VERBAL REHEARSAL –

SAYING & DOING

TRACING

Use Print-out

WRITING STARTS WITH ALGORISMS

FOLLOW THE ARROWS WHILE TALKING

1	ONE STROKE. $27,990
START 2	AROUND, DOWN TO THE LEFT AND THE FOOT.
3 (1, 2, 3)	ZIG-ZAG AND AROUND.
4 (1, 2, 3)	SLIPPERY DIP, RIGHT, DOWN.
5 (1, 2, 3)	READY FOR THE NEXT NUMBER SLANT, AROUND AND THE TOP.
6 (1, 2)	AROUND AND AROUND.
7 (1, 2)	RIGHT, CURVE DOWN.
8 (1, 2)	SLOPPY NUMBERS CREATE PROBLEMS BIG EGG, SMALL EGG.
9 (1, 2, 3)	AROUND AND WE ALL FALL DOWN. NOT → 9 1 9
10	ONE STROKE, ONE JELLYBEAN.

THE 9 IS A CURVED NUMBER, NOT THE SILLY 'SCHOOLSTICK'. THE 9 IS LIKE THIS! THERE CANNOT POSSIBLY BE A MIX-UP!

→ PRACTISE TRACING EVERY LESSON UNTIL ALGORISMS
-3 MINUTES-

TALLY DRAW & COUNT

|	1					卌 卌 (10) |	11				
||	1	2				卌 卌 (10) ||	11	12			
|||	1	2	3			卌 卌 (10) |||	11	12	13		
||||	1	2	3	4		卌 卌 (10) ||||	11	12	13	14	
卌 START	1	2	3	4	5	卌 卌 (10) 卌	11	12	13	14	15
卌 |	1 6	2	3	4	5	卌 卌 (10) 卌 |	11 16	12	13	14	15
卌 ||	1 6	2 7	3	4	5	卌 卌 (10) 卌 ||	11 16	12 17	13	14	15
卌 |||	1 6	2 7	3 8	4	5	卌 卌 (10) 卌 |||	11 16	12 17	13 18	14	15
卌 ||||	1 6	2 7	3 8	4 9	5	卌 卌 (10) 卌 ||||	11 16	12 17	13 18	14 19	15
卌 卌	1 6	2 7	3 8	4 9	5 10	卌 卌 (10) 卌 卌	11 16	12 17	13 18	14 19	15 20

COUNTING IN TWOS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

ODD AND EVEN NUMBERS

COUNTING NUMBERS

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
222	333	444	555	666	777	888	999	1000 ONE THOUSAND	

POINT TO 74, 15, 23, ETC.

POINT TO NUMBER BEFORE AND AFTER 99, ETC.

ODD (3 LETTERS) NUMBERS END IN 1, 3, 5, 7, 9.

EVEN (4 LETTERS) NUMBERS END IN 0, 2, 4, 6, 8.

POINT & SAY ODD (OR EVEN) BEFORE (AFTER) 67 (44), ETC.

BEWARE

IN DUTCH, ALL NUMBERS ARE READ
BACK TO FRONT

16	27
TIEN ZES	TWINTIG EN ZEVEN (START)

IN ENGLISH, ONLY THE TENS.

16	19
TEEN SIX	TEEN NINE (START)

EXERCISE TO STOP **MIXING UP**

16 AND 61

DOES IT START WITH A ONE ?

YES: SIXTEEN **NO** SIXTY-ONE

REPEAT THIS EXERCISE WITH OTHERS TILL THE PROBLEM IS SOLVED.

2+15 17

KEEP LOOKING AT THE PROBLEM,

SEE NOT AT THE CEILING! **SAY**

STOP THIS COMMON HABIT IMMEDIATELY.

ODD OR NUMBER AWARENESS EVEN

1. AN EXERCISE IN SEEING.
2. AN AID TO FORMULATE GENERAL RULES.

1	15	8	56	27	3
77	98	4	73	37	68
9	12	58	85	103	666
887	554	72	94	49	2
10	81	52	60	74	20
101	104	92	21	12	11
25	54	45	86	19	91
41	14	63	36	999	1000

GENERAL RULES FOR ADDITION

-NO MORE TABLES!

IF YOU ADD 1 TO A NUMBER, YOU GET THE NEXT NUMBER.

IF YOUR FATHER CHANGES PLACES WITH YOUR MOTHER, DO YOU STILL RECOGNISE YOUR PARENTS? OF COURSE YOU DO!

∴ THEREFORE

YOU MUST **SEE** THAT $3+1=1+3$

ANSWERS ONLY! MEANS: DO NOT REPEAT THE QUESTIONS.
EYES & BRAIN OPERATE AT THE SPEED OF LIGHT, ONE MILLION TIMES FASTER THAN SOUND!

WHAT'S YOUR NAME? JOHN.	15+1? SIXTEEN.

1+14	7+1	1+16	8+1	18+1	1+9
19+1	1+2	13+1	1+5	12+1	1+3
11+1	4+1	1+10	17+1	1+6	15+1
27+1	1+36	49+1	1+50	61+1	1+72
83+1	1+94	99+1	66+1	1+77	999+1

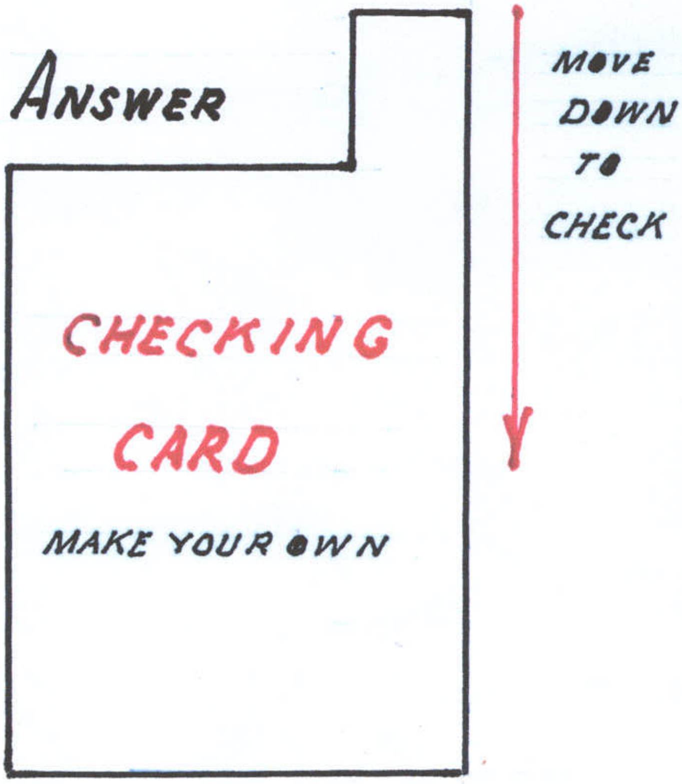
CHECKING ANSWERS

MANY CHILDREN HAVE BACK PROBLEMS BECAUSE MATHS BOOKS GET THICKER AND THICKER. TO SOLVE THE PROBLEM, COMMERCIAL ENTERPRISE DESIGNED BIGGER BAGS!

MOST TEACHERS SELECT EITHER ODD OR EVEN SUMS, SO HALF THE BOOK IS WASTED.

SINCE ONLY A COUPLE OF PAGES ARE USED PER LESSON, IT'S EXTREMELY STUPID TO CARRY THE WHOLE BOOK TO SCHOOL ONLY BECAUSE THE ANSWERS ARE AT THE BACK!

-NUMBERS IN A NUTSHELL-
IS ON CD SO THAT YOU CAN USE PRINT-OUTS.



1 + 99	100
63 + 1	64
1 + 44	45
79 + 1	80
1 + 19	20

- NO MORE TABLES -

GENERAL RULES FOR ADDITION

IF YOU ADD 2 TO AN ODD NUMBER,
YOU GET THE NEXT ODD NUMBER

2 + 5	7	2 + 15	17	2 + 3	5
2 + 13	15	9 + 2	11	2 + 19	21
39 + 2	41	2 + 93	95	99 + 2	101
2 + 221	223	333 + 2	335	89 + 2	91

IF YOU ADD 2 TO AN EVEN NUMBER,
YOU GET THE NEXT EVEN NUMBER.

2 + 4	6	14 + 2	16	6 + 2	8
2 + 16	18	18 + 2	20	2 + 38	40
50 + 2	52	2 + 108	110	98 + 2	100
2 + 332	334	444 + 2	446	2 + 88	90

THE 9-PARTNERS

PROFESSIONAL MEMORY TRAINING: THE KEY TO LEARNING IS INTEREST.
POSITIVE REINFORCEMENT: 'REMEMBER' INSTEAD OF 'DON'T FORGET'!

SAY ALOUD, "I WANT TO REMEMBER" THEIR SUM IS NINE		SAY ALOUD "HOW CAN I REMEMBER?" BY USING VERBAL REHEARSAL
1	8	THE SKINNY ONE GOES WITH THE FAT ONE
2	7	2 LOOKS A BIT LIKE 7, EXCEPT FOR THE FOOT
3	6	3 GOES WITH ITS DOUBLE 6 GOES WITH ITS HALF
5	4	5 AND 4 ARE NEXT DOOR

TEACHER-STUDENT EXERCISES:

1. I SAY 4, YOU SAY 5.
   I SAY 5, YOU SAY 4, ETC.
2. IN ORDER: 1,8 - 8,1 - ETC.
3. RANDOM ORDER (QUICK):
   2,7 - 1,8 - 4,5 - 6,3 - 3,6, ETC.

MAGIC 9 - PARTNERS IN ACTION

ECONOMY LEARNING	NUMBERS IN BULK	
EYES LOOK AND SEE	WITH THE SPEED OF LIGHT	
1 + 8	11 + 8	COVER 19
ANSWERS	ONLY!	
3 + 6	23 + 6	29
2 + 7	32 + 7	39
4 + 5	44 + 5	49
8 + 1	58 + 1	59
7 + 2	67 + 2	69
6 + 3	76 + 3	
5 + 4	YOU NEED A RUN TO JUMP. START SEVENTY ...... NINE	

ADDITION -NO MORE TABLES-

MAGIC 9 - RECIPE

8 FOR THE PRICE OF ONE!

THINK ONE LESS AND TEEN

7 + 9 6 TEEN ← COVER UP	9 + 9 → 8 TEEN
5 + 9 4 TEEN	8 + 9 7 TEEN
6 + 9 FIF TEEN NOT 5 TEEN	4 + 9 THIR TEEN NOT 3 TEEN
3 + 9 TWELVE NOT 2 TEEN	2 + 9 ELEVEN NOT 1 TEEN

ADDITION WITH MAGIC 9 OUTSIDE

THINK 1 MORE → 1 LESS

EYES "TRAVEL" AT 300 000 km/s	ANSWERS ONLY — VOICE 300 m/s
11 + 9 — 20	76 + 9 — EIGHTY.... 5
EYES NOTICE	NUMBERS IN BULK
23 + 9 — 32	87 + 9 — 96
LEARN ONE RECIPE,	DO 5 MILLION SUMS.
32 + 9 — 41	95 + 9 — 104
49 + 9 — 58	104 + 9 — 113
54 + 9 — 63	232 + 9 — START — TWO HUNDRED AND FORTY... 1
68 + 9 — 77	AWARENESS. ALERTNESS. BRAINCELLS IN ACTION

ADDITION WITH MAGIC

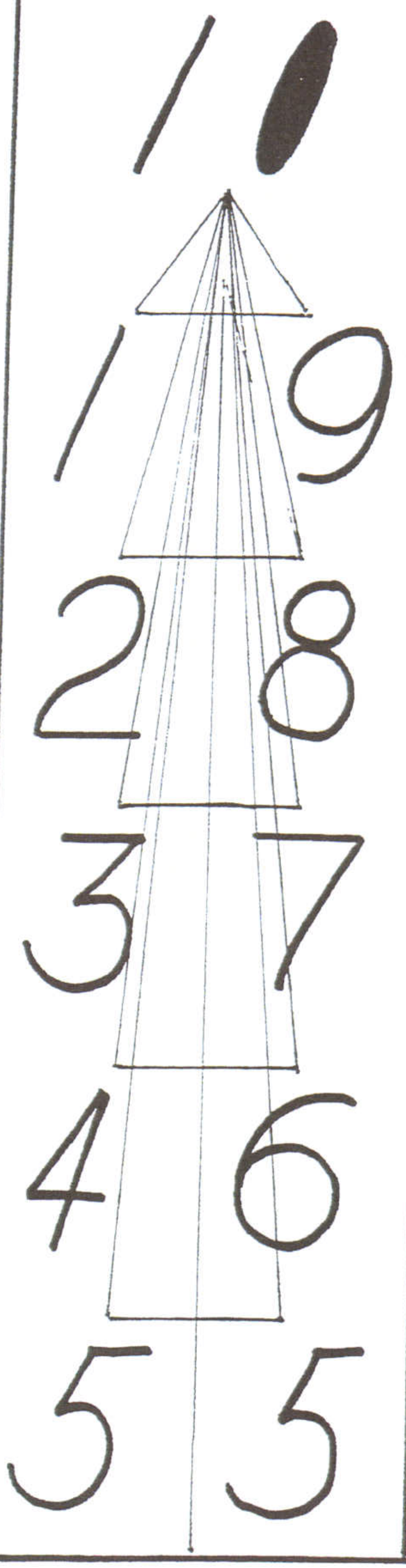
9 INSIDE

ALL PURPOSE RECIPE

THINK 1 MORE 1 LESS

ANSWERS ONLY 19 + 2 21	79 + 7 EIGHTY 6
EYES NOTICE → 29 + 4 33	ONE GLANCE IS ENOUGH 89 + 9 98
39 + 6 45	99 + 2 10 101
49 + 8 57	119 + 3 12 122
59 + 3 62	129 + 5 13 4
69 + 5 74	COUNTING FINGERS? CALCULATOR? DON'T BE RIDICULOUS!

THE TEN-PARTNERS



EXERCISES: POINT AND SAY

CHOPSTICK

FOLLOW THE TRACKS

1 . 9 . 10	2 . 8 . 10
3 . 7 . 10	4 . 6 . 10
5 . 5 . 10	5 . 5 . 10
6 . 4 . 10	7 . 3 . 10
8 . 2 . 10	9 . 1 . 10

LISTEN

TEN	TAKE	ONE NUMBER	NINE BEFORE
TEN	TAKE	TWO EVEN	EIGHT BEFORE
TEN	TAKE	3 → THREE	7 SEVEN
TEN	TAKE	FOUR NEXT	SIX EVEN
TEN	TAKE	ONE HAND	THE OTHER HAND

MULTIPLES OF 10 NO MORE TABLES

THINK 1 MORE AND ZERO

ANSWERS ONLY		EYES	
14 + 6 (10)	20	196 + 4	200
21 + 9 (10)	30	283 + 7	290
35 + 5 (10)	40	379 + 1	380
42 + 8 (10)	50	468 + 2	470
57 + 3 (10)	60	557 + 3	560
68 + 2 (10)	70	642 + 8	650
79 + 1 (10)	80	735 + 5	740
83 + 7 (10)	90	821 + 9	830
96 + 4 (10)	100	EYES NOTICE 914 + 6	920

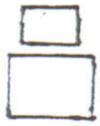
ADDITION NO MORE TABLES

THE NUMBER AND TEEN

ADD 1 — KEEP

9 + 10	19	14 + 10 (2 4)	24
6 + 10	16	51 + 10	61
8 + 10	18	22 + 10	32
4 + 10	14	39 + 10	49
7 + 10	17	77 + 10	87
NOT 5 TEEN 5 FIF + 10 TEEN	15	99 + 10 (10)	109
NOT 3 TEEN 3 THIR + 10 TEEN	13	188 + 10 (19 8)	198
NOT 1 TEEN 1 + 10 ELEVEN	11	515 + 10	525
NOT 2 TEEN 2 + 10 TWELVE	12	990 + 10 (100)	1000

DUTCH DUBBEL	# DOUBLES	FRENCH DOUBLE

3 + 3	6	THINK, THEN SEE → 3
4 + 4	8	 FOUR + FOUR 1234 5678
5 + 5	10	2 HANDS. 10 FINGERS
6 + 6	12	1 2 DOZEN, SIXES DOUZE (FRENCH)
7 + 7	14	2 WEEKS OF 7 DAYS FORTNIGHT FOURTEEN NIGHTS
8 + 8	16	2 EIGHTS SIXTEEN DOUBLES ARE ALWAYS EVEN

USING STEPPING STONES

DOUBLES PLUS ONE

FROM THE KNOWN	TO THE UNKNOWN	
IF 3 'n 3 IS 6, THEN	3 'n 4's 1 MORE	7
IF 5 'n 5 IS 10, THEN	5 'n 6's 1 MORE	11
IF 6 'n 6 IS 12, THEN	6 'n 7's 1 MORE	13
IF 7 'n 7 IS 14, THEN	7 'n 8's 1 MORE	15

HALVES

EVENS

6	3
14	7
2	1
8	4
18	9
4	2
10	5
16	8
12	6
20	10

ODDS

MENTALLY

13 - 1
THINK 1 LESS

12

6 ADD $\frac{1}{2}$

5	$2\frac{1}{2}$
9	$4\frac{1}{2}$
15	$7\frac{1}{2}$
7	$3\frac{1}{2}$
3	$1\frac{1}{2}$
11	$5\frac{1}{2}$

ADDITION SPECIALS

– NO MORE TABLES – 3 SENSES AT WORK!

FOUR SEVEN

ELEVEN

SAY ALOUD, HEAR AND SMELL EAU DE COLOGNE (USE IT!)

IF 4+7=11	IF 4+7=11	IF 4+7=11
5+7=12	4+8=12	3+8=11

EVEN DOWN

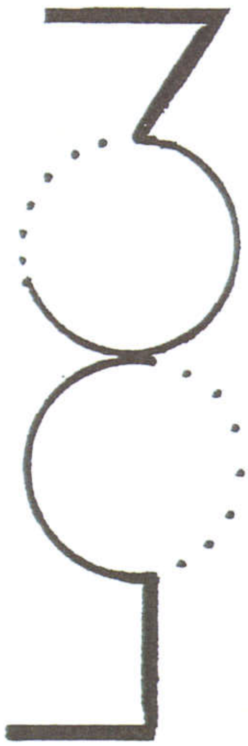
8

6

4 TEEN

3+4=7

CREATIVE NUMERACY



SAY ALOUD 3 5 8

5 8 13

TASK BASED EXERCISES

PROCEDURE

STEP 1.	SORTING USING INSTRUCTIONS.
STEP 2.	SORTING FROM MEMORY.
STEP 3.	STUDENTS TEST THEMSELVES BY USING "THE BOARD" (PAGE IN THE BOOK). ANSWERS SHOULD BE COVERED AND CHECKED AS THEY GO.

ATTENTION

1.	LESS HASTE, MORE SPEED! / FESTINA LENTE.
2.	EACH STEP HAS TO BE REPEATED UNTIL IT CAN BE DONE TO PERFECTION. ∴ NO INTERMEDIATE MARKING!
3.	ONLY WHEN STEP 3 YIELDS 100%, SHOULD STUDENTS START DOING ALGORISMS!

TASK BASED RECOGNITION EXERCISES

IMPORTANT REMINDER

Initially, the newly-discovered patterns didn't provide the hoped-for advantage.
It took more than one year to become aware of the reason.
I had mentioned that learning numberfacts is no more difficult than learning the names of fellow students. HOWEVER, BEFORE being able to call their names, ONE HAS TO RECOGNISE THEM FIRST.
SIMILARLY, students must first cut out the cards and sort them according to the groups mentioned.
Answers must only be learnt when this TASK-BASED exercise can be done with the greatest of ease.
It is planned looking AND seeing.

WITHOUT THE ANSWERS!
CUT OUT COPY NEXT PAGE. (THE BOARD) AT THE BACK OF THE BOOK. CARDS FIT IN A MATCH BOX.

MIX AND MATCH

1.	THE 9-PARTNERS	7.	DOUBLES +1
2.	THE 10-PARTNERS	8.	X + 9 A NUMBER + 9
3.	X + 1 MEANS: A NUMBER + 1	9.	X + TEEN A NUMBER + TEN
4.	2 + EVEN	10.	4+7, 3+8 / 5+7, 4+8
5.	2 + ODD	11.	8 + 6
6.	THE DOUBLES	12.	3+5 / 5+8

9	
1	8
2	7
3	6
5	4
10	
ONE	NINE
TWO	EIGHT
THREE	SEVEN
FOUR NEXT	SIX EVEN
5 2 HANDS	5

X+1	
3+1	4
2+1	3
5+1	6
4+1	5
7+1	8
6+1	7
10+1	11
2+EVEN	
2+6	8
2+10	12
2+4	6
2+ODD	
2+5	7
2+9	11
2+3	5
DOUBLES	
4+4=2X4	

10+10	20
3+3	6
1+1	2
4+4	8
7+7	14
2+2	4
5+5	10
8+8	16
6+6	12
DOUBLES+1	
3+4	7
6+7	13
5+6	11
7+8	15
X+9	
3+9	12
5+9	14
4+9	13

SORTING	
6+9	15
8+9	17
7+9	16
9+9	18
X+TEEN	
3+10	13
5+10	15
7+10	17
9+10	19
8+10	18
4+10	14
4+7	11
3+8	11
5+7	12
4+8	12
8+6	14
3+5 5+8	8 13

EXTENSION ADDITION FACTS 1.

A PICTURE SPEAKS 1000 WORDS!

FLUENCY IN MENTAL ARITHMETIC

20	10+10	11+9	12+8

20 PLUS	LARGEST 20+9=29

SEE

19+9

SEE

START SAYING

18

TWENTY-----EIGHT

EVENTUALLY TWENTY-EIGHT

SEE

18+7

SEE

START SAYING

15

TWENTY----FIVE

EVENTUALLY TWENTY-FIVE

THE PHILOSOPHY

STARTING IS A VISUAL REACTION: SEE 1, SAY TWENTY
IT COULD NEVER BE THIRTY!

IT WILL MOTIVATE THE STUDENT TO CONTINUE, THUS DEFUSING THE "I DON'T KNOW" SYNDROME,

AN ARTIFICIALLY CREATED PROBLEM, THE RESULT OF EMPHASISING UNDERSTANDING INSTEAD OF DOING.

USE A CARD TO COVER ANSWERS

20 → 45 POSSIBILITIES ← AND 20 PLUS

12 + 9	16 + 4	15 + 7	16 + 6	17 + 3
21	20	22	22	20
15 + 9	16 + 8	14 + 6	17 + 6	18 + 2
24	24	20	23	20
17 + 4	18 + 7	12 + 8	18 + 9	17 + 8
21	25	20	27	25
13 + 7	17 + 5	18 + 6	19 + 7	19 + 3
20	22	24	26	22
19 + 9	18 + 8	17 + 7	15 + 6	18 + 5
28	26	24	21	23
16 + 9	18 + 4	19 + 5	11 + 9	14 + 8
25	22	24	20	22
16 + 7	17 + 9	15 + 8	14 + 7	13 + 9
23	26	23	21	22
19 + 4	16 + 5	13 + 8	19 + 6	18 + 3
23	21	21	25	21
14 + 9	19 + 2	19 + 8	15 + 5	19 + 1
23	21	27	20	20

EXTENSION

A PICTURE SPEAKS 1000 WORDS

ADDITION FACTS 2.

SEE

$$24 + 8$$

SEE: 12

SAY

THIRTY----TWO

QUICKER AND QUICKER

EVENTUALLY IN 1 SEC.

SEE

$$45 + 9$$

SEE: 14

SAY

FIFTY----FOUR

QUICKER AND QUICKER

EVENTUALLY IN 1 SECOND!

SEE

$$67 + 6$$

SEE: 13

SAY

SEVENTY---THREE

QUICKER AND QUICKER

EVENTUALLY IN 1 SECOND

SEE

$$88 + 3$$

SEE: 11

SAY

NINETY---ONE

QUICKER AND QUICKER

EVENTUALLY IN 1 SECOND

MULTIPLES OF 10 (USE CHECK CARD)		54+8 SIXTY...		YOU KNOW THAT 4+8 = 12		THINK 1 MORE, SO START BY SAYING TWO	
21+9	30	23+8	31	26+5	31	37+8	45
42+8	50	33+9	42	46+7	53	58+3	61
33+7	40	44+7	51	36+6	42	47+9	56
64+6	70	54+8	62	56+8	64	68+4	72
55+5	60	64+9	73	77+4	81	88+6	94
86+4	90	85+7	92	66+9	75	78+5	83
77+3	80	75+6	81	97+6	103	98+7	105
98+2	100	105+9	114	87+5	92	38+9	47
89+1	90	95+8	103	27+7	34	INCREASE 28 BY 8 28+8	36

YOU'LL NEED THIS TO ADD A COLUMN OF NUMBERS

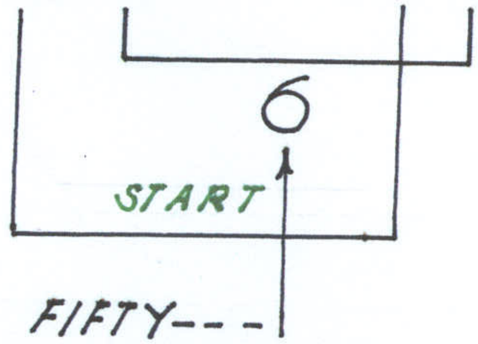
PRINT OUT: COVER UP ANSWERS, THEN CHECK

MENTALS EXTENSION

TYPE 1.

SEE SAY

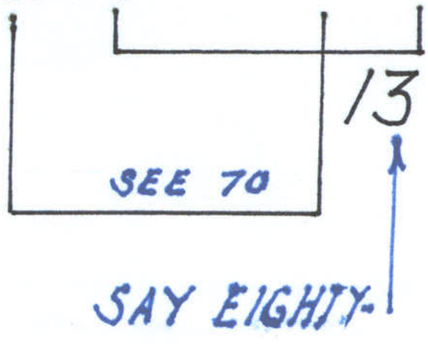
21 + 35 = 56



TYPE 2.

SEE SAY

34 + 49 = 83



CHECK

MENTALS

Extension Test

12+1 9	31	17+1 5	32	13+1 8	31	16+1 7	33
26+1 5	41	27+1 8	45	28+1 9	47	29+1 6	45
27+2 9	56	28+2 8	56	29+2 2	51	28+2 7	55
36+1 6	52	35+1 7	52	39+1 8	57	38+1 5	53
38+2 4	62	36+2 8	64	39+2 4	63	37+2 7	64
46+1 9	65	45+1 6	61	44+1 7	61	49+1 7	66
47+2 6	73	45+2 9	74	49+2 3	72	44+2 8	72
53+3 9	92	57+3 4	91	58+3 6	94	59+3 5	94
59+4 9	108	58+4 3	101	55+4 8	103	54+4 9	103

ALGORISM

MEANS:

$$34 + 52 =$$

CORRECTLY READ AS

POSITIVE 34, POSITIVE 52
LEFT OUT

AND WRITTEN AS

$$\begin{array}{r} 34 \\ +52 \\ \hline \end{array}$$

ANSWER — ONLY ONE LINE!

POSITIVE 52 ∴ THE + SYMBOL IS IN FRONT OF 52, NOT BEHIND 34!

∴ SYMBOL MEANS THEREFORE

ALGORISMS

TYPE 1.

START

TRACING →

34
+ 52
86

EYES 'MOVE' DOWN!

ADDING UP IS A SILLY EXPRESSION AND HABIT BECAUSE THE ANSWER IS WRITTEN UNDERNEATH.

PEN WRITES

11 POSSIBILITIES

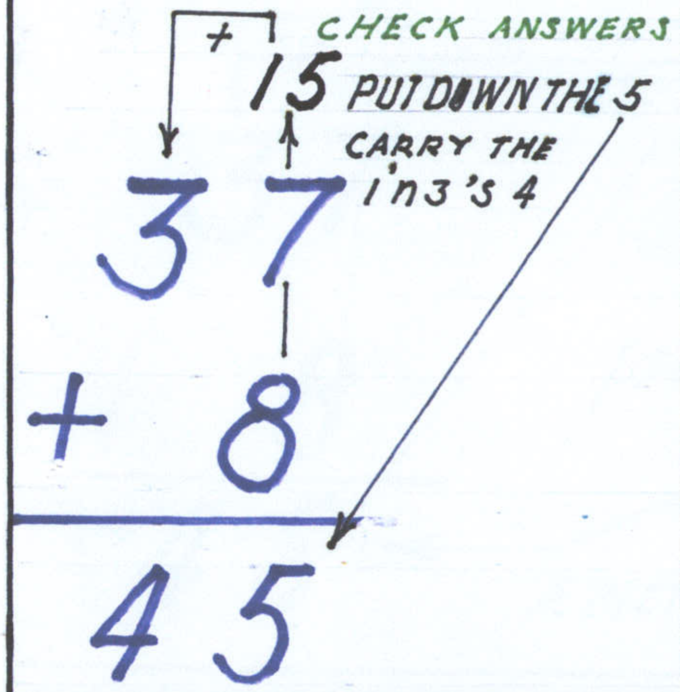
ANSWER, CHECK, WRITE

1.	2	3	4	5	6
11 + 21	13 + 41	15 + 61	17 + 81	22 + 32	22 + 54
22 + 76	33 + 54	33 + 66	33 + 33	44 + 54	COVER UP ANSWERS WHILE WORKING, THEN CHECK.

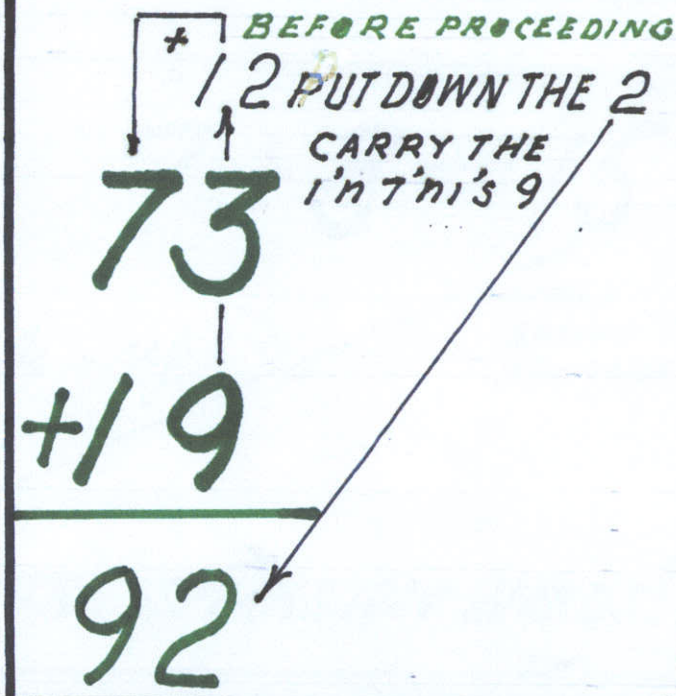
32	98	54	87	76	99	98	66	54	98	76	

ALGORISM DIAGRAMS

TYPE 2. | STAGE 1. | USING VERBAL REHEARSAL AND ANSWER SHEET TO ESTABLISH ROUTINE.



NOTE: ADDING UP TO SUIT THE DIAGRAM



INVENT YOUR OWN

FLUENCY IS OBTAINED BY USING SHORT HAND.

'n INSTEAD OF AND OR PLUS
's INSTEAD OF IS OR EQUALS

7'n 8's 15, PUT DOWN THE 5,
CARRY THE 1'n 3's 4.
DO NOT PUT IT DOWN!

NOTE: CARRY THE 1'n 3. LIKE IN RELAY RACES, THE BATON IS NOT PUT DOWN BUT CARRIED TO THE NEXT RUNNER;
WITH 2 NUMBERS, IT'S ALWAYS 1, BECAUSE 9+9=18

THIS ROUTINE WAS INVENTED TO DEFUSE THE "I DON'T UNDERSTAND STUDENTS"

STUDENTS JUST KEEP REPEATING WHAT THE TEACHER SAYS UNTIL THEY CAN DO IT BY THEMSELVES. THAT'S HOW ADVERTISING WORKS

→ IF YOU EXPLAIN TOO MUCH, YOU EXPLAIN NOTHING!

ALGORISMS

TYPE 2.	STAGE 2.	DIRECT WITH VERBAL REHEARSAL

PRACTISE ON WHITE BOARD THEN CHECK.

$$\begin{array}{r} 78 \\ +\ 5 \\ \hline 83 \end{array}$$

8'n 5's 13,

PUT DOWN THE 3,

CARRY THE 1'n 7's 8

NO CRUTCH FIGURE!

25 POSSIBILITIES
DO THE FOLLOWING COMBINATIONS BY USING EITHER STAGE 1 OR 2

EXERCISE THE BRAIN CELLS - NOT THE PEN - OTHERWISE THEY BOTH PERISH.

19 +1	58 +6	37 +5	16 +4	29 +3	18 +2	27 +4	26 +5	15 +5
39 +4	59 +6	68 +7	47 +6	17 +3	19 +2	48 +5	78 +8	89 +9
36 +6	49 +5	38 +4	69 +7	28 +3	79 +8	57 +7	31 86	20 98
20 43 42	64 65 54	42 75 42	20 53 76	32 20 31	20 21 87	31 53 64	STAGE 3. DIRECT WRITTEN STAGE 4 AS MENTALS	

ALGORISMS

TYPE 3. DIRECT WITH VERBAL REHEARSAL

$$\begin{array}{r} 43 \\ +29 \\ \hline 72 \end{array}$$

3'n 9's 12,

PUT DOWN THE 2,

CARRY THE 1'n 4's 5

'n 2's 7

INVENT & CHECK YOUR OWN (CALCULATOR). WITH OR WITHOUT VR
THEN DO TYPE 4.

TYPE 4.

$$\begin{array}{r} 64 \\ +88 \\ \hline 152 \end{array}$$

INVENT AND CHECK YOUR OWN

NUMBER FLUENCY

EXERCISES

REDUCING A NUMBER TO ONE DIGIT

KEEP ADDING THE DIGITS

13 (4)	56 (11, 2)	88 (16, 7)

9 OR 9-COMBINATIONS: IGNORE 592718 (32, 5) ! SHORT CUT

THE 9-REMAINDER

IS THE SUM OF THE DIGITS!

$$21 = (2 \times 9) + 3$$

CHECKING ALGORISMS: TRACHTENBERG METHOD

21	3
+37	+1
58	4
13 → 4	✓

SUCCESS RATE 98%

IF YOU HAD WRITTEN 49, THE MISTAKE WOULDN'T HAVE BEEN DETECTED.

NUMEROLOGY: TO FIND RULING NUMBER: 20.12.1928 (7)

ALGORISMS

	CHECK
123	6
+456 (IGNORE)	6
579	3 ✓

12
3

IGNORE

FIRST CANCEL THE NINES

WHEN CHECKING

1234	1
+5678	8
6912	0 ✓

'ROLL UP AS YOU GO'

FROM NOW ON, ONLY USE YOUR EYES.
IT'S SILLY TO SAY WHAT YOU SEE;
LIGHT IS 1MILLIONS X FASTER!

379	1
523	1
+417	3
1319	5 ✓

5

NOTE: WITH 3 NUMBERS, YOU ONLY CARRY 1 OR 2 BECAUSE 3×9=27

PALINDROMIC NUMBERS

START WITH ANY NUMBER,
ADD THE REVERSE,
KEEP GOING UNTIL...

```
  379
+ 973
-----
 1352
+2531
-----
 3883
```

IT READS THE SAME BOTH WAYS.

DOING ALGORISMS

1. IS A THERAPEUTIC ACTIVITY.
IF YOU DON'T USE BRAINCELLS, THEY DIE OFF!
THE DANGER OF A COMPUTER SOCIETY.

2. HANDY WHEN CALCULATOR BATTERIES PERISH.

ADDING A SINGLE COLUMN
STAGE 1.

EYES

SAY ALOUD

3	
5	
6	8
7	14
8	21
9	29
2	38
	40

8 ← TOUCH WITH PEN IF YOU LIKE

ADDING 2 COLUMNS
STAGE 2. MENTALLY

EYES ONLY.
PROFESSIONAL ROUTINE.
NO CRUTCH FIGURE!

THINK			THINK
10	8	3	
17	7	5	8
23	6	6	14
28	5	7	21
	3	8	THINK 29
	31	9	PUT 9

CARRY THE 2 'n
DO NOT PUT IT DOWN,
IT DISTURBS FLUENCY!

CHECKING ALGORISMS FOR ADDITION

NUMBERS ARE REDUCED TO 1 DIGIT AS YOU GO.

EXAMPLE: 2+3+4 (9 IGNORE) +5+6 (2) +4 WRITE 6

119	2	123	6	321	6
120	3	456	6	654	6
121	4	789	6	987	6
+ 122	5	+ 101	2	+ 101	2
482	5 ✓	1469	2 ✓	2063	2 ✓

1234567	1	9101112	6
7654321	1	8131415	5
+ 1234567	1	+ 1617189	6
10,123,455	✓ 3	18,849,716	✓ 8

THERE IS NO LIMIT. YOU DON'T HAVE TO SAY THE ANSWERS!

USE COMFORTABLE SIZE NUMBERS ON WHITE BOARD
DO NOT USE SILLY MINI SQUARES; THEY RUIN YOUR HANDWRITING

— NO MORE TABLES —

GENERAL RULES FOR SUBTRACTION

IF YOU TAKE ONE FROM A NUMBER, YOU GET THE NUMBER BEFORE.

20-1=19	43-1=42	11-1=10

THE DIFFERENCE BETWEEN TWO CONSECUTIVE NUMBERS IS ONE.

17-16=1	15-14=1	26-25=1

THE DIFFERENCE BETWEEN TWO CONSECUTIVE ODDS OR EVENS IS TWO.

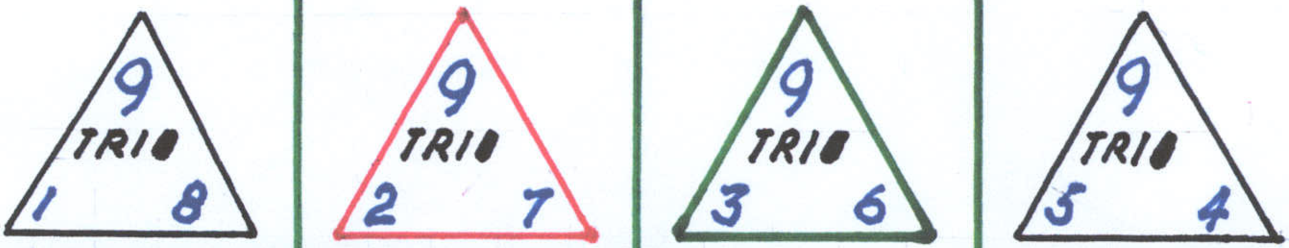
19-17=2	14-12=2	7-5=2	6-4=2

IF YOU TAKE 2 FROM AN ODD OR EVEN NUMBER, YOU GET THE ODD OR EVEN NUMBER BEFORE.

15-2=13	8-2=6	27-2=25	12-2=10

-NO MORE TABLES-

THE 9-PARTNERS IN ACTION



1 + 8 = 9	2 + 7 = 9	3 + 6 = 9	5 + 4 = 9
9 − 1 = 8	9 − 2 = 7	9 − 3 = 6	9 − 4 = 5
9 − 8 = 1	9 − 7 = 2	9 − 6 = 3	9 − 5 = 4

REPLACE THE 9 BY THE MISSING PARTNER

19 − 1	29 − 3	39 − 2	49 − 4
18	26	37	4 45
59 − 8	69 − 7	79 − 6	89 − 5
51	62	73	84

CREATIVE NUMERACY

"THE BRAIN DOESN'T ABSORB PURE DATA; IT HAS TO BE SEEN THROUGH THE SPECTACLES OF AN IDEA." (EDWARD DE BONO)

NUMBERS IN A NUTSHELL IS THE ONLY COURSE IN THE WORLD THAT COMBINES THE ABOVE KNOWLEDGE WITH PRINCIPLES OF PROFESSIONAL MEMORY TRAINING. EYES INSTEAD OF INTELLECT.

- NO MORE TABLES -

10 FOR THE PRICE OF ONE

NOBODY HAS EVER NOTICED IT BEFORE

ONLY WITH 1 IN FRONT → 13 − 9 = 4

ADD SEE

THE REVERSE: 13 − 4 = 9

SINCE 1+3=4 ANSWER

THE EYES	ANSWERS ONLY		HAVE IT !
15−9 (6)	11−9 (2)	16−7 (9)	16−9 (7)
11−2 (9)	18−9 (9)	14−9 (5)	15−6 (9)
10−1 (9)	19−10 (9)	13−4 (9)	18−9 (9)
13−9 (4)	10−9 (1)	14−5 (9)	12−9 (3)
17−8 (9)	17−9 (8)	19−9 (10)	12−3 (9)

THINK 1 LESS, 1 MORE

65 − 9

56 ← SAY

SEE

20 − 9	37 − 9	41 − 9	53 − 9
11	28	32	44
68 − 9	72 − 9	84 − 9	106 − 9
59	63	75	97

THE MISSING 9 - PARTNER

9 − 1 8	9 − 3 6	9 − 2 7	9 − 4 5
9 − 8 1	9 − 7 2	9 − 6 3	9 − 5 4

REPLACE THE 9 BY THE PARTNER

19 − 1 18	29 − 3 26	39 − 2 37	49 − 4 45
59 − 8 51	69 − 7 62	79 − 6 73	219 − 5 214

THE 10-PARTNERS

10 TRIO 9 1	10 TRIO 8 2	10 TRIO 7 3	10 TRIO 6 4	10 TRIO 5 5
NiNe oNe	EIGHT Two	sEvEn thrEE	NEXT EVEN	2 HANDS
9+1	8+2	7+3	6+4	5+5
1+9	2+8	3+7	6+4	5+5

FIND	THE	MISSING	10 -	PARTNER
10−7	10−2	10−4	10−8	10−9
3	8	6	2	1
10−5	10−6	10−3	40−4	60−6
5	4	7	36	54

18−10	75−10	34−10	83−10
8	65	24	73
92−10	61−10	56−10	47−10
82	51	46	37

THE HALVES

16 – 8	20 – 10	2 – 1	18 – 9	10 – 5
14 – 7	6 – 3	8 – 4	4 – 2	12 – 6
46 – 23	62 – 31	88 – 44	24 – 12	40 – 20

HALF 52	HALF 74	HALF 96
HALF 40 20	HALF 60 30	HALF 80 40
HALF 12 + 6	HALF 14 + 7	HALF 16 + 8
26	37	48
CALCULATOR	HALF 234	234 ÷ 2 = 117
3 4 7	3 5 8	4 7 11
7 - 3 / 7 - 4	8 - 3 / 8 - 5	11 - 4 / 11 - 7
14 – 8	12 – 7	13 – 5
FROM 8 TO 10 2	FROM 7 TO 10 3	FROM 5 TO 10 5
+ 4	+ 2	+ 3
EVENTUALLY JUST 6	EVENTUALLY JUST 5	EVENTUALLY JUST 8

20 – 9 11	53 – 6 47	31 – 8 23	42 – 7 35	64 – 5 59
75 – 6 69	86 – 7 79	98 – 9 89	32 – 5 27	44 – 6 38

MENTALS
SELECT 1 METHOD ONLY.

53 − 6 THINK: 50−3 = 47	42 − 7 THINK: FROM 7 TO 40 = 33 + 2 = 35
32 − 5 THINK: 30 − 3 = 27	31 − 8 THINK: 22+1 = 23

GIVING CHANGE
WITHOUT CALCULATOR IS QUICKER

THE 100 − PARTNERS

10	90	90	10
20	80	80	20
30	70	70	30
40	60	60	40
50	50	50	50

THE 90 − PARTNERS

10	80	80	10
20	70	70	20
30	60	60	30
40	50	50	40

100 − 73
THINK: 7+20 = 27

100 − 44
THINK: 6+50 = 56

90 − 16
THINK: 4+70 = 74

TASK BASED EXERCISES

PROCEDURE

STEP 1.	SORTING USING INSTRUCTIONS.
STEP 2.	SORTING FROM MEMORY.
STEP 3.	STUDENTS TEST THEMSELVES BY USING "THE BOARD" (PAGE IN THE BOOK). ANSWERS SHOULD BE COVERED AND CHECKED AS THEY GO.

ATTENTION

1.	LESS HASTE, MORE SPEED! FESTINA LENTE.
2.	EACH STEP HAS TO BE REPEATED UNTIL IT CAN BE DONE TO PERFECTION. ∴ NO INTERMEDIATE MARKING!
3.	ONLY WHEN STEP 3 YIELDS 100%, SHOULD STUDENTS START DOING ALGORISMS!

TASK BASED RECOGNITION EXERCISES

SORT ACCORDING TO THE FOLLOWING
GENERAL RULES AND RECIPES

CUT OUT THE CARDS (BACK OF THE BOOK) WITHOUT ANSWERS.
THE ONES ON THE 'BOARD' (PAGE IN BOOK) ACT AS
AN ATTENTION DIRECTOR!
STORE THE CARDS IN A MATCH BOX (REVISION)

1.	X-1 MEANS: A NUMBER-1	7.	HALVES
2.	DIFFERENCE TWO CONSECUTIVE NUMBERS	8.	THE 9-PARTNERS
3.	DIFFERENCE TWO EVEN NUMBERS	9.	ADD THE DIGITS
4.	DIFFERENCE TWO ODD NUMBERS	10.	THE DIGITS - THEIR SUM
5.	EVEN-2	11.	ALGORISM NECESSITIES 12-8 \ FROM 8-10=2 ADD 2
6.	ODD-2	12.	7-3 7-4 | 8-5 8-3

SORTING				12-6	6	13-9	4	13-5	8
$x-1$		CONSECUTIVE ODD		11-5	5	15-9	6	11-6	4+1
8-1	7	7-5	2	9 PARTNER		14-9	5	13-7	6
2-1	1	5-3	2	9-8	1	DIGITS – THEIR SUM		14-6	8
7-1	6	EVEN-2		9-6	3	18-9	9	13-6	7
3-1	2	8-2	6	9-4	5	17-8	9	11-7	4
6-1	5	4-2	2	9-2	7	16-7	9	12-7	3+2
4-1	3	6-2	4	9-5	4	15-6	9	15-7	8
5-1	4	ODD-2		9-1	8	14-5	9	11-8	2+1
DIFFERENCE 2 CONSECUTIVES		7-2	5	9-7	2	13-4	9	15-8	7
8-7	1	3-2	1	9-3	6	12-3	9	12-8	4
4-3	1	5-2	3	ADD THE DIGITS		11-2	9	14-8	6
7-6	1	HALVES		18-9	9	11-3	7+1	13-8	5
5-4	1	16-8	8	11-9	2	11-4	6+1	7-3	4
CONSECUTIVE EVEN		6-3	3	17-9	8	12-4	6+2	8-5	3
8-6	2	14-7	7	12-9	3	11-5	5+1	7-4	3
6-4	2	8-4	4	16-9	7	12-5	5+2	8-3	5

ALGORISMS

TYPE 1.

98 NEGATIVE → −55 43	76 −44 32
POSITIVE 98 NEGATIVE 55	(POS.) 76 NEG. 44 = (POS) 32

NEGATIVE NUMBERS • POSITIVE NUMBERS

"TAKE AWAY" IS ONLY FOR FISH & CHIPS OR HAMBURGERS!

YEAR 7: $-7+5-3+4-2+1=-2$

THINK: THERE ARE 12 NEGATIVES

THERE ARE 10 POSITIVES

THE NEGATIVES ARE "WINNING BY" 2

REGULAR CHECK-UP

$12-3=9$

$12=3+9$

TYPE 2.

BEFORE ATTEMPTING THIS TYPE, SUBTRACTIONS GIVEN ON THE NEXT PAGE MUST BE KNOWN WITHOUT ANY HESITATION

56	11(2) ✓	SHORT WAY WITH 9 REMAINDER
−21	+3	
35	8	

$35+21=56$

LONG WAY

AT LEAST IT'S FINITE!

45 (4+5=9) POSSIBILITIES

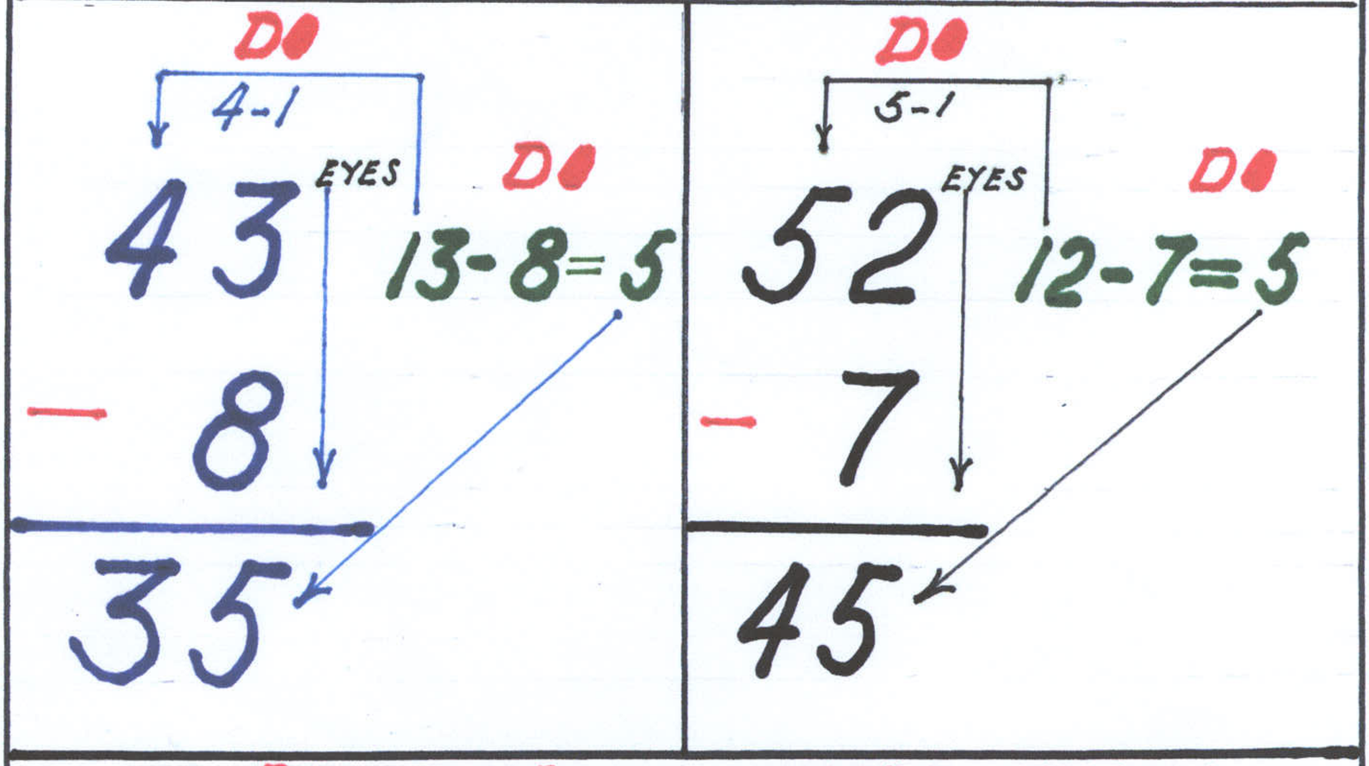
1.		2.		3.		4.		5	
10 − 1	9	11 − 3	8	12 − 5	7	13 − 7	6	14 − 9	5
11 − 2	9	12 − 3	9	13 − 8	5	14 − 5	9	10 − 2	8
15 − 9	6	10 − 7	3	11 − 5	6	12 − 6	6	10 − 4	6
11 − 8	3	12 − 9	3	13 − 5	8	14 − 7	7	16 − 9	7
10 − 5	5	11 − 6	5	12 − 8	4	13 − 6	7	14 − 8	6
11 − 7	4	10 − 8	2	16 − 8	8	15 − 7	8	13 − 9	4
10 − 3	7	11 − 4	7	12 − 7	5	13 − 4	9	16 − 7	9
11 − 9	2	12 − 4	8	10 − 9	1	15 − 8	7	10 − 6	4
14 − 6	8	15 − 6	9	17 − 8	9	18 − 9	9	17 − 9	8

A PICTURE SPEAKS A 1000 WORDS

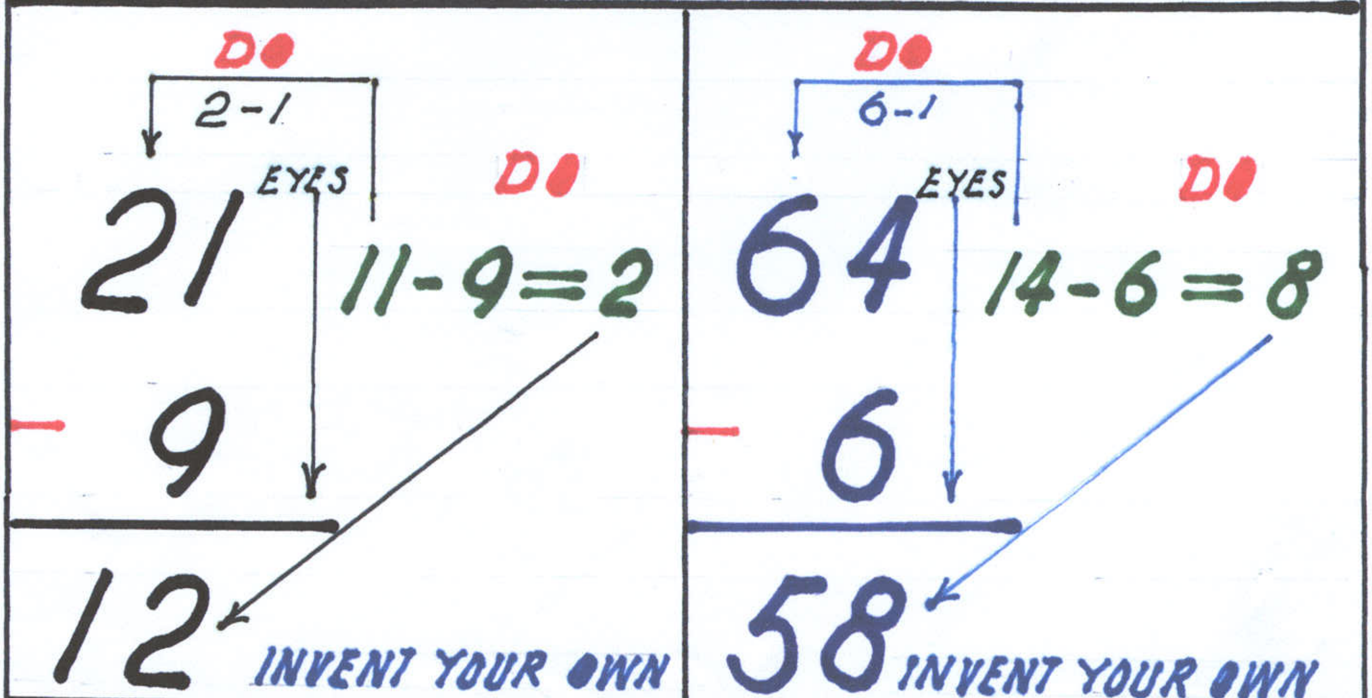
ALGORISMS TYPE 2.

ROUTINE

NO EXPLANATIONS. NO EXAMPLES WITH FIGURES CROSSED OUT! STUDENTS WILL SWITCH OFF BEFORE THE TEACHER STARTS.



POSITIVE INSTRUCTIONS



THIS METHOD WAS INVENTED TO DEFUSE 'I DON'T UNDERSTAND' STUDENTS.

ALGORISMS TYPE 3

DO

5-1-1

EYES

DO

$$54 - 15 = 39$$

14-5=9

DO

7-1-2

EYES

DO

$$73 - 25 = 48$$

13-5=8

POSITIVE INSTRUCTIONS

DO

9-1-3

EYES

DO

$$195 - 36 = 159$$

15-6=9

DO

3-1-1

12-7=5

EYES

DO

$$333 - 178 = 155$$

13-8=5

INVENT & CHECK YOUR OWN

CALCULATOR OR 9-REM.

SUBTRACTION ALGORISMS

ALGORISMS ALL SORTS

YOU CAN ONLY SOLVE A PROBLEM WHEN YOU RECOGNISE IT!

82	10✓	123	6✓	456	6✓	2468	2✓
−16	7	−49	4	−178	7	−1609	7
66	3	74	2	278	8	859	4

		0 BECOMES 9	
70203	3✓	500001	6✓
−44444	2	−187345	1
25759	1	312656	5

9876543	6✓	64444444	7✓
−7090806	3	−55555555	4
2785737	3	8888889	3

DOING ALGORISMS IS A VISUAL ACTIVITY, BUT IT IS DIFFERENT FROM WATCHING T.V. BECAUSE THE BRAIN IS ALSO INVOLVED

THE NUMBER OF POSSIBLE SUBTRACTIONS IS INFINITE. IT IS EXTREMELY STUPID AND COSTLY TO FILL BOOKS WITH THEM.

18 FACTS	NO MORE TABLES	INSTEAD OF 36!

MULTIPLICATION

FAST ADDITION	4X5 = 5+5+5+5=5X4

INTELLIGENT NUMERACY

GENERAL RULES

ANYTHING X 0=0		4 MEANS 4X1	IIII
TIMES 10	ADD 0	6X10=60	23X10=230
TIMES 100	ADD 00	17X100=1700	

THE RECIPE: 9 TRIO 1 8	THE RECIPE: 9 TRIO 2 7	THE RECIPE: 9 TRIO 3 6	THE RECIPE: 9 TRIO 5 4

THINK ONE LESS AND THE PARTNER — NO FINGERS

2X9	5X9	4X9	8X9
18	45	36	72
7X9	9X9	3X9	6X9
63	81	27	54

— CREATIVE NUMERACY —

MULTIPLICATION RECIPES

COPYING TABLES IS A CHORE AND THEREFORE AS USELESS AS TRYING TO SELL ICECREAM ON THE MOON.

6 AND EVEN — HALF THE NUMBER & THE NUMBER

4X6	6X2	6X6	8X6
24	12	36	48

5 AND EVEN — HALF THE NUMBER & ZERO

4X5	5X2	5X6	8X5
20	10	30	40

5 AND ODD — ANSWER ENDS IN 5

5X5	3X5	5X7
25	15	35
YOU SEE 2 FIVES	ODD BEFORE 3	ODD BEFORE 5

— CREATIVE NUMERACY —

UNIQUE NUMBER PATTERNS

3 PAIRS

$12 = 3 \times 4$	$56 = 7 \times 8$

7	8	8	7
3	4	8	6
ONE	DOWN	EVEN	DOWN
2	3	6	4
1	2	4	2

A PICTURE SPEAKS 1000 WORDS

8

3

2   3   4

TWENTY — FOUR

CREATIVE NUMERACY

LOOKING FOR WAYS TO REMEMBER

3X3

● ● ●

● ● ●

● ● ●

CHANNEL 9

NINE PARTNERS

1 8
2 7
3 × 6
4 5

3X6
18

FIRST
THIRD

2 FOURS 8
4 FOURS 16
6 TEEN

2 EIGHTS
SIXTEEN
DOUBLES ARE EVEN

DOUBLES

2X3=3+3
ETC...

→ IMAGINATION ←

2 X 7

2 WEEKS OF 7 DAYS: FORTNIGHT
=
14 NIGHTS

1 WEEK 1 WEEK
7 X 7
A 'FORTNINE' 49

4X7

4 WEEKS OF 7 DAYS

THE PERFECT MONTH

FEBRUARY

28 DAYS

2ND MONTH, 8 LETTERS!

TASK BASED EXERCISES

PROCEDURE

STEP 1.	SORTING USING INSTRUCTIONS.
STEP 2.	SORTING FROM MEMORY.
STEP 3.	STUDENTS TEST THEMSELVES BY USING "THE BOARD" (PAGE IN THE BOOK). ANSWERS SHOULD BE COVERED AND CHECKED AS THEY GO.

ATTENTION

1.	LESS HASTE, MORE SPEED! FESTINA LENTE.
2.	EACH STEP HAS TO BE REPEATED UNTIL IT CAN BE DONE TO PERFECTION. ∴ NO INTERMEDIATE MARKING!
3.	ONLY WHEN STEP 3 YIELDS 100%, SHOULD STUDENTS START DOING ALGORISMS!

TASK BASED RECOGNITION EXERCISES

- CUT OUT COPY OF NEXT PAGE (THE BOARD), WITHOUT THE ANSWERS; IT'S AT THE BACK OF THE BOOK.

- MIX & MATCH. (INDIRECT LEARNING.)

  WHEN STUDENTS PLACE A CARD ON THE BOARD, THE ANSWER WILL CATCH THEIR ATTENTION.

SORTING INSTRUCTIONS

x STANDS FOR A NUMBER

1.	10 x x — EERST ALLE MEANS 10X1, 10X2, ...	8.	6 x EVEN
2.	9 x x	9.	3 x 3
3.	3x4 & 7x8	10.	3 x 6
4.	7x3 & 8x4	11.	7x6 & 8x8
5.	8 x 3	12.	4x7 & 7x7
6.	5 x EVEN (1 2 3 4)	13.	4 x 4
7.	5 x 000 (1 2 3)	14.	2x2, 2x3, 2X4, 2X7, 2x8

TASK BASED RECOGNITION EXERCISES

10X1	10	9X2	18	2X2	4	10X6	60
9X4	36	2X3	6	10X7	70	2X4	8
6X2	12	10X3	30	9X3	27	3X4	12
4X7	28	6X4	24	8X4	32	10X2	20
5X2	10	9X5	45	6X6	36	7X7	49
7X3	21	5X5	25	9X7	63	6X8	48
10X4	40	2X7	14	12X11	121	3X3	9
8X3	24	10X5	50	5X6	30	9X6	54
5X4	20	4X4	16	9X9	81	10X8	80
10X9	90	9X8	72	10X10	100	5X8	40
3X5	15	3X6	18	2X8	16	5X7	35
7X6	42	7X8	56	8X8	64	23X100	2300

ALGORISMS TYPE 1.

123	123	212	111
X 2	X 3	X 4	X 5
246	369	848	555

THEY EACH MULTIPLY THE TOP

```
    123
  X 12   START
  -----
    246
```

THE SECOND STARTS IN SECOND PLACE

```
 +123▮
  -----
  1,476
```

TYPE 2

TYPE 3

```
    123
  X 321   START
  -----
    123
   246▮
```

THE THIRD STARTS IN THIRD PLACE

```
  369▮▮
  -----
  39,483
```

THIRTY-NINE THOUSAND

FOUR HUNDRED AND EIGHTY-THREE

USE ANSWER SHEET

ALGORISM DIAGRAMS

CHECK ANSWERS BEFORE PROCEEDING.

+

18 24 PUT DOWN THE 4
CARRY THE 2 'N 18'S 20

34
X6

204

TYPE 4

+

3.
24 36 PUT DOWN THE 6
2. 1.
CARRY THE 3 'N 24'S 27

69
X4

276

STAGE 1.

SAY ALOUD

+

56 40 PUT DOWN THE 0
CARRY THE 4 'N 56'S 60

75
X8

600

+

10 30 PUT DOWN THE 0
CARRY THE 3 'N 10'S 13

26
X5

130

ALGORISMS
THE PROFESSIONAL WAY.

123456789	123456789
X 2	X 3
246913578	370370367

'ROLL UP' AS YOU GO

13579	7	SO FAR 3 24680	2
X 46	X 1	X 57	X 3
81474		172760	
543160		1234000	
624634	7 ✓	1406760	6 ✓
SO FAR 3		SO FAR 2	

THE FIRST 3 MULTIPLES OF ...

2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9
4	6	8	10	12	14	16	18
6	9	12	15	18	21	24	27

SYMBOL ÷ DIVISION ÷

DIVISION IS THE OPPOSITE OF MULTIPLICATION.

6X2=2X6 BUT 6÷2 ≠ 2÷6

↑ IS NOT

6X2 =12	∴ 12÷2=6 AND
TRIO	12÷6=2

NOTE: ALL 36 MULTIPLICATION MUST BE KNOWN WITHOUT ANY FALSE STARTS OR HESITATION IN 36 SECONDS OR LESS!

		ANSWER		SHEET 1.		20 SECONDS	
10÷1	10	10÷10	1	9÷1	9	9÷9	1
8÷1	8	8÷8	1	7÷1	7	7÷7	1
6÷1	6	6÷6	1	5÷1	5	5÷5	1
4÷1	4	4÷4	1	3÷1	3	3÷3	1
2÷1	2	2÷2	1	20÷2	10	20÷10	2
COVER		ANSWERS		THEN		CHECK	

÷ DIVISION ÷

ANSWER SHEET 2.

18÷2	9	14÷2	7	10÷5	2	16÷8	2
12÷6	2	16÷2	8	18÷9	2	12÷2	6
10÷2	5	14÷7	2	24÷4	6	36÷6	6
48÷6	8	24÷6	4	48÷8	6	20÷4	5
40÷5	8	30÷6	5	40÷8	5	30÷5	6
20÷5	4	35÷5	7	25÷5	5	15÷5	3
35÷7	5	15÷3	5	21÷3	7	32÷8	4
64÷8	8	21÷7	3	32÷4	8	42÷7	6
42÷6	7	24÷3	8	9÷3	3	24÷8	3
18÷3	6	16÷4	4	18÷6	3	28÷4	7
49÷7	7	28÷7	4	COVER UP ANSWERS, THEN CHECK		TIME	42 SEC

÷ DIVISION ÷

ANSWER SHEET 3

36÷4	9	4÷2	2	70÷10	7	56÷8	7
8÷2	4	45÷5	9	63÷9	7	30÷10	3
36÷9	4	12÷3	4	27÷3	9	81÷9	9
50÷5	10	70÷7	10	56÷7	8	40÷10	4
63÷7	9	12÷4	3	8÷4	2	72÷9	8
60÷6	10	45÷9	5	6÷3	2	27÷9	3
72÷8	9	54÷9	6	60÷10	6	80÷8	10
6÷2	3	90÷10	9	54÷6	9	30÷3	10
80÷10	8	100÷10	10	90÷9	10	50÷10	5
90÷9	10	40÷4	10	COVER UP ANSWERS, THEN CHECK		TIME	38 SEC.

WAYS TO REMEMBER: NEXT PAGE

DIVISION CLUES

9- RECIPE IN REVERSE. VISUALISE	$27 \div 9$ | 3 +1	$18 \div 9$ | 2 +1
$45 \div 9$ | 5 +1	$72 \div 9$ | 8 +1	$54 \div 9$ | 6 +1
$36 \div 9$ | 4 +1	$81 \div 9$ | 9 +1	$63 \div 9$ | 7 +1
SAME NUMBERS, DIFFERENT PLACE	$27 \div 3$ | 9 +1	$18 \div 2$ | 9 +1

DIVISION CLUES

45 ÷ 5 | 9 (+1)	72 ÷ 8 | 9 (+1)	54 ÷ 6 | 9 (+1)
36 ÷ 4 | 9 (+1)	81 ÷ 9 | 9 (+1)	63 ÷ 7 | 9 (+1)

THE 9-PARTNERS

1	8
2	7
3	6
5	4

CREATIVE NUMERACY

18 ÷ 3   18 ÷ 6

VISUALISE

DIVISION CLUES

6 AND EVEN RECIPE IN REVERSE

ANSWER ↓	ANSWER ↓	↓ ANSWER	↓ ANSWER
24 ÷ 6	36 ÷ 6	12 ÷ 6	48 ÷ 6
HALF	HALF	HALF	HALF
WHOLE	WHOLE	WHOLE	WHOLE

6	6	6	6
24 ÷ 4	36 ÷ 6	12 ÷ 2	48 ÷ 8
HALF	HALF	HALF	HALF
WHOLE WHOLE	WHOLE WHOLE	WHOLE WHOLE	WHOLE WHOLE

5 AND EVEN RECIPE IN REVERSE

DOUBLE → 4	DOUBLE → 8	DOUBLE → 6	DOUBLE → 2
20 ÷ 5	40 ÷ 5	30 ÷ 5	10 ÷ 5

5	5	5	5
20 ÷ 4	40 ÷ 8	30 ÷ 6	10 ÷ 2
DOUBLE	DOUBLE	DOUBLE	DOUBLE

5 AND ODD RECIPE IN REVERSE	35 ÷ 7 | 5 (CONSEC. ODD)	15 ÷ 3 | 5 (CONSEC. ODD)

VISUALISING IS PART OF PROFESSIONAL MEMORY TRAINING

25 ÷ 5 | 5	35 ÷ 5 | 7 (CONSECUTIVE ODDS)	15 ÷ 5 | 3 (CONSECUTIVE ODDS)

12 ÷ 3 | 4	12 ÷ 4 | 3

CONSECUTIVE NUMBERS

56 ÷ 7 | 8	56 ÷ 8 | 7

24 ÷ 3	8	8 ×
24 ÷ 8	3	2 3 4

WITH CREATIVE NUMERACY DEAD NUMBERS BECOME ALIVE

7 3 21	21 ÷ 3 | 7 21 ÷ 7 | 3
MULTIPLICATION	DIVISION
8 4 32	32 ÷ 4 | 8 32 ÷ 8 | 4
MULTIPLICATION	DIVISION
7 6 42	42 ÷ 6 | 7 42 ÷ 7 | 6
MULTIPLICATION	DIVISION
8 8 64	64 ÷ 8 | 8

FORTNIGHT	"FORTNINE"	FEBRUARY
14 ÷ 7 = 2	49 ÷ 7 = 7	28 ÷ 7 = 4

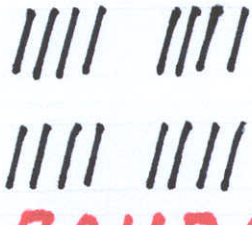
CHANNEL

9

$9 \div 3 = 3$

2 EIGHTS
SIXTEEN

16



4 FOURS

$16 \div 4 = 4$

$8 \div 8 = 1$ $8 \div 1 = 8$

PROTO TYPES

$61 \div 6 = 11$ $61 \div 11 = 6$

THE HALVES: $8 \div 2$ MEANS HALF EIGHT

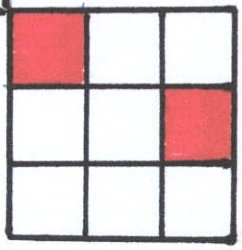
$20 \div 2$	10	$20 \div 10$	2	$10 \div 2$	5	$10 \div 5$	2
$18 \div 2$	9	$18 \div 9$	2	$8 \div 2$	4	$8 \div 4$	2
$16 \div 2$	8	$16 \div 8$	2	$6 \div 2$	3	$6 \div 3$	2
$14 \div 2$	7	$14 \div 7$	2	$4 \div 2$	2	$4 \div 2$	2
$12 \div 2$	6	$12 \div 6$	2	$2 \div 2$	1	$2 \div 1$	2

TRUE STORY: HOW MANY HALVES IN ONE? YEAR 10 STUDENT: "STUFFED IF I KNOW!"

16÷4 4	9÷3 3	25÷5 5	49÷7 7	QUICK ANSWERS			1 MIN. TEST
ANOTHER	STUDENT	SHOULD	CHECK	THE	ANSWERS	GIVEN.	
10÷10 1	4÷1 4	14÷2 7	30÷3 10	80÷10 8	54÷6 9	28÷4 7	24÷3 8
8÷8 1	2÷1 2	10÷2 5	50÷5 10	30÷10 3	72÷8 9	15÷5 3	21÷7 3
3÷3 1	9÷1 9	12÷2 6	80÷8 10	70÷10 7	81÷9 9	35÷7 5	24÷8 3
7÷7 1	7÷1 7	6÷3 2	60÷6 10	72÷9 8	63÷7 9	15÷3 5	21÷3 7
9÷9 1	5÷1 5	8÷4 2	70÷7 10	36÷9 4	40÷5 8	24÷4 6	32÷4 8
6÷6 1	3÷1 3	10÷5 2	90÷9 10	45÷9 5	20÷5 4	36÷6 6	56÷7 8
4÷4 1	20÷2 10	16÷8 2	50÷10 5	63÷9 7	30÷5 6	48÷8 6	12÷3 4
2÷2 1	4÷2 2	14÷7 2	90÷10 9	27÷9 3	20÷4 5	24÷6 4	32÷8 4
5÷5 1	18÷2 9	12÷6 2	60÷10 6	54÷9 6	30÷6 5	48÷6 8	18÷6 3
10÷1 10	6÷2 3	18÷9 2	20÷10 2	45÷5 9	40÷8 5	64÷8 8	42÷7 6
8÷1 8	16÷2 8	20÷2 10	100÷10 10	36÷4 9	28÷7 4	56÷8 7	18÷3 6
6÷1 6	8÷2 4	40÷4 10	40÷10 4	27÷3 9	35÷5 7	12÷4 3	42÷6 7

DIVISION INVOLVES FRACTIONS

1.



THERE ARE 9 SQUARES
2 OF THEM ARE RED WRITTEN AS
$\frac{2}{9}$ (TWO NINTHS)
NOT 2 OUT OF 9 !

16 ÷ 3 BY CALCULATOR (16 a^b/c 3) WILL SHOW $5\frac{1}{3}$
(FIVE AND ('N, NOT PLUS) ONE THIRD) NOT 5 AND ONE OUT OF 3
AND NOT 5 R1 (REMAINDER 1) EITHER

2. THE ÷ SYMBOL IS LIKE A VACANT FRACTION

3. EQUIVALENT (SAME VALUE) FRACTIONS $\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$ ETC.

4. FRACTIONS MUST BE SIMPLIFIED $\frac{7}{28} = \frac{1}{4}$

5. LONG DIVISION : 527 ÷ 3 IS LEFT AS IS !

NOT $3 \overline{)527}$ with 175 R2 written above

A. IT LOOKS CLUMSILY COMPLICATED

B. STUDENTS WILL CHANGE 3 ÷ 57 TO $3 \overline{)57}$
BECAUSE THEY'RE USED TO SEEING THE SMALLER NUMBER FIRST. ∴ DO NOT USE 'INTO' EITHER!

LONG DIVISION SHORT

WITH THE ADVENT OF THE CALCULATOR, EXERCISES ARE ONLY THERAPEUTIC. THEY IMPROVE NUMBER FLUENCY, PROVIDED THEY ARE DONE THE OLD-FASHIONED

PROFESSIONAL WAY!

NOT UMPTEEN DIFFERENT WAYS TO SUIT INNUMERATE STUDENTS (NEW MATHS!)

STEP 1. LEAVE IT THIS WAY NORMAL PLACE

DEMONSTRATION ONLY

$$527 \div 3 = 175\frac{2}{3}$$

FOR THE ANSWER

STEP 2. STEP 3

$1 \times 3 =$ 3

DEMONSTRATION ONLY

SUBTRACT

22

MAXIMUM MULTIPLE

$7 \times 3 =$ 21

BY USING THE MAXIMUM MULTIPLE, THERE IS ONLY ONE WAY

SUBTRACT

17

MAXIMUM MULTIPLE

$5 \times 3 =$ 15

SUBTRACT

2

NOTE

1. LONG DIVISION WITHOUT STEP 2.
2. SHORT DIVISION WITHOUT STEPS 2 & 3

LONG DIVISION TYPE 1.

FRACTIONAL ANSWER	DECIMAL ANSWER

Fractional answer

$34813 \div 5 = 6962\frac{3}{5}$

30
48
45
31
30
13 DEMONSTRATION ONLY
10
3 DEMONSTRATION ONLY

ATTENTION DIRECTOR

Decimal answer

$= 6962.6$

THINK ONLY

34813.000......

BEFORE USING 0, INSERT (DP) DECIMAL POINT

30
30
0

INVENT YOUR OWN.

USE CALCULATOR TO CHECK ANSWERS

LONG DIVISION TYPE 1.

$$246810 \div 4 = 61702\frac{2}{4}$$

24

DO NOT WRITE 0 6

4

28

28

10

8

2

CALCULATOR 61702.5
AS A DECIMAL

1. SIMPLIFIED

$$61702\frac{1}{2}$$

2. 2 IS NOT A MULTIPLE OR GREATER THAN A MULTIPLE OF 4, ∴ TAKE ON THE NEXT DIGIT AS WELL.

DO NOT START WITH 0! THE ANSWER IS A QUANTITY, NOT A SERIAL NUMBER.

3. 1 IS NOT A MULTIPLE OR GREATER THAN A MULTIPLE OF 4. NOW, YOU DO WRITE 0 BEFORE USING THE LAST DIGIT; IN THIS CASE 0.

ALL POSSIBILITIES HAVE NOW BEEN DEALT WITH.
SILLY, COMPLICATED SCHOLASTIC HABITS ARE IGNORED.
FIRST PEOPLE FORM HABITS, THEN THE HABITS FORM THE PEOPLE.

NO MORE TABLES? ONE SPECIAL EXCEPTION!

TABLE PRODUCTION

WHEN DIVIDING A NUMBER BY A 2-DIGIT ONE, YOU MAY LIKE TO USE THE TRACHTENBERG METHOD INVENTED WHILE IN A CONCENTRATION CAMP DURING W.W II

THE TABLES ARE PRODUCED BY WRITING A NUMBER ON A PIECE OF PAPER AND HOLDING IT ABOVE THE SAME NUMBER WRITTEN DOWN. ADD THE TWO WHICH WILL GIVE THE 2ND MULTIPLE. MOVE THE PAPER DOWN. ADD THE NUMBER TO THE 2ND MULTIPLE TO GET THE 3RD, ETC...

THE 10TH ONE IS THE CHECKING ONE!

13 ON A PIECE OF PAPER		14	↓	16	↓	17	↓	23	↓
13	1.	14	1.	16	1.	17	1.	23	1.
26	2.	28	2.	32	2.	34	2.	46	2.
39	3.	42	3.	48	3.	51	3.	69	3.
52	4.	56	4.	64	4.	68	4.	92	4.
65	5.	70	5.	80	5.	85	5	115	5.
78	6.	84	6.	96	6.	102	6.	138	6.
91	7.	98	7.	112	7.	119	7.	161	7.
104	8.	112	8.	128	8	136	8.	184	8.
117	9.	126	9.	144	9	153	9.	207	9.
130✓	10.	140✓	10.	160✓	10	170✓	10.	230✓	10.

LONG DIVISION

TRACHTENBERG METHOD TYPE 2.

987,654,321 ÷ 13

91
77
65
126
117
95
91
44
39
53
52 ↓ PUT 0 BEFORE USING THE 2
121
117
4

13	1.	75973409 $\frac{4}{13}$ USE THE
26	2.	LARGEST
39	3.	**MULTIPLE** OF 13 EQUAL
52	4.	OR LESS THAN
65	5.	THE NUMBER SHOWN, JUST
78	6.	SCAN DOWN THE
91	7.	COLUMN.
104	8	
117	9.	
130 ✓	10	CHECK! 10×13=130. START

REMINDER: FOUR THIRTEENTH

ANSWER

75,973,409 $\frac{4}{13}$

DIVISIBILITY

$12 \div 2 = 6$	$2 \mid 12$	2 DIVIDES 12	$5 \mid 13575$ ↑
$12 \div 3 = 4$	$3 \mid 12$	3 DIVIDES 12	
$12 \div 4 = 3$	$4 \mid 12$	4 DIVIDES 12	$5 \mid 47890$ ↑
$12 \div 6 = 2$	$6 \mid 12$	6 DIVIDES 12	
	$5 \nmid 12$	5 DOES NOT DIVIDE 12	

A NUMBER IS DIVISIBLE BY 2 IF IT'S EVEN

64 - 32 - 16 - 8	BY 4 IF IT'S EVEN AGAIN BY 8 IF IT'S STILL EVEN.

A NUMBER IS DIVISIBLE BY 3 IF THE DIGIT-SUM IS DIVISIBLE BY 3.

$3 \mid 744$ BECAUSE $3 \mid 15$ AND $3 \mid 6$

IF THE QUOTIENT IS EVEN, IT IS ALSO DIVISIBLE BY 6. $744 \div 3 = 246$ WHICH IS EVEN.

$9 \mid 123456789$

BECAUSE THE SUM OF THE DIGITS IS DIVISIBLE BY 9

$11 \mid 84579$ BECAUSE

$(8+5+9)-(4+7)=11$

ODD POSITION — EVEN POSITION

ALGEBRA THE PROFESSIONAL WAY

$7 + \square = 12$ IS NOT ALGEBRA!

1. IT INVOLVES COUNTING FINGERS OR GUESSING.
2. THIS SO-CALLED NEW MATHS WILL CREATE PROBLEMS IN YEAR 7 BECAUSE THEY HAVE TO FIRST GET RID OF A BAD HABIT BEFORE LEARNING TO APPLY THE LAW OF OPPOSITES.

"WHAT DO I HAVE TO ADD TO 7 TO GET 12" IS COUNTERPRODUCTIVE.

3. IF YOU WANT TO INTRODUCE ALGEBRA, "MYSTERY NUMBERS" ARE REPRESENTED BY LETTERS – $a, b, c, \cdots x, y, z$, NOT BOXES!

$6+3=9$ $\therefore$	$6=9-3$ $3=9-6$	4. SIMPLE EQUATIONS 4.-WHY NUMBER SENTENCES FIRST-ARE DONE MENTALLY, NOT THE SCHOLASTIC INCOMPETENT WAY!
$7-4=3$	$7=3+4$	$4a$ IS SHORT FOR $4 \times a$ 4 APPLES, NOT 4 TIMES 1 APPLE!
$3 \times 2=6$	$3=6 \div 2$	$4a=12$ | $a=3$ YOU ONLY THINK, NOT WRITE $12 \div 4$
$\frac{8}{4}=2$	$8=2 \times 4$	$\frac{x}{5}=4$ | $x=20$
$2x+2=6$ CAN'T BE SO WHY	DONE ON FINGERS THE BOXES?	MENTALLY $x=2$

CONVERSIONS

FRACTION TO DECIMAL	FRACTION TO PERCENTAGE	DECIMAL TO PERCENTAGE
$\frac{3}{4}$ MEANS	$\frac{5}{6}=$	$.075=$
$3\div 4=.75$	DO $5\div .06\doteq$ APPROXIMATELY 83.3%	FROM D —TWO PLACES→ P 7.5%

DECIMAL TO FRACTION	PERCENTAGE TO FRACTION	PERCENTAGE TO DECIMAL
$2.013=2\frac{13}{1000}$	$3\%=\frac{3}{100}$	D ←TWO PLACES— P FROM
	$3\frac{1}{2}\%=\frac{7}{200}$	$140\%=$ THINK DP
$2.014=2\frac{7}{500}$	$3\frac{1}{3}\%=\frac{1}{30}$	1.4

CONVERSIONS		RELATIONS FAMOUS DOZEN	
SIMPLE FRACTIONS	DECIMAL FRACTIONS	DECIMALS ← 2 TO →	PERCENTAGE
$3\frac{4}{5}$	$3\frac{8}{10}$	3.8	380%
$1\frac{1}{4}$	$1\frac{25}{100}$	1.25	125%
$2\frac{3}{4}$	$2\frac{75}{100}$	2.75	275%
$\frac{7}{100}$	$\frac{7}{100}$	.07	7%
$\frac{9}{10}$	$\frac{9}{10}$	.9	90%
$\frac{11}{20}$	$\frac{55}{100}$	.55	55%
$\frac{12}{25}$	$\frac{48}{100}$	.48	48%
$\frac{13}{50}$	$\frac{26}{100}$	.26	26%
$\frac{1}{125}$	$\frac{8}{1000}$	.008	.8%
$\frac{7}{200}$	$\frac{35}{1000}$	.035	3.5%

$2+\frac{7}{10}+\frac{9}{100}+\frac{3}{1000}=2.793$ $\quad$ $23+\frac{7}{100}+\frac{9}{10000}=23.0709$

LOOK AND SEE HOW IT WORKS

A SYSTEM SHOULD BE APPLIED, NOT CONTINUOUSLY EXPLAINED! RED MEANS STOP; DON'T EXPLAIN

DECIMAL SYSTEM
READING & WRITING

COUNTING NUMBERS	←→ 2-WAY TRAFFIC
1	ONE
COVER UP ONE SIDE TO PRACTISE THE OTHER 21	TWENTY-ONE
321	THREE HUNDRED AND TWENTY-ONE
4,321	FOUR THOUSAND, THREE HUNDRED AND TWENTY-ONE
54,321	FIFTY-FOUR THOUSAND, THREE HUNDRED AND TWENTY-ONE.
654,321	SIX HUNDRED AND FIFTY-FOUR THOUSAND, THREE HUNDRED AND TWENTY-ONE
7,654,321	SEVEN MILLION, SIX HUNDRED AND FIFTY-FOUR THOUSAND. THREE HUNDRED AND TWENTY-ONE.
87,654,321	EIGHTY-SEVEN MILLION, SIX HUNDRED AND FIFTY-FOUR THOUSAND, THREE HUNDRED AND TWENTY-ONE.
987,654,321	NINE HUNDRED AND EIGHTY-SEVEN MILLION, SIX HUNDRED AND FIFTY-FOUR THOUSAND, THREE HUNDRED AND TWENTY-ONE.

DECIMAL SYSTEM

COUNTING NUMBERS

←——→ 2-WAY TRAFFIC

301	THREE HUNDRED AND ONE
4,001	FOUR THOUSAND AND ONE
5,032	FIVE THOUSAND AND THIRTY-TWO
56,006	FIFTY-SIX THOUSAND AND SIX
608,045	SIX HUNDRED AND EIGHT THOUSAND AND FORTY-FIVE
7,000,019	SEVEN MILLION AND NINETEEN
88,050,060	EIGHTY-EIGHT MILLION FIFTY-THOUSAND AND SIXTY.
900600300	NINE HUNDRED MILLION, SIX HUNDRED THOUSAND, THREE HUNDRED.
249000	1000 LESS THEN 1/4 MILLION

ROUNDING OFF		HALF WAY AND ABOVE — UP
15	NEAREST 10	20
267	NEAREST 100	300
1500	NEAREST 1000	2000

DECIMALS

THE CLUMSY WAY $5+\frac{7}{10}+\frac{2}{100}+\frac{3}{1000}+\frac{7}{10000}$	$3+\frac{5}{100}+\frac{3}{10000}$
THE ECONOMICAL WAY 5.7237	VACANCIES 3.0503
$5.7237=$	$3000+600+50+4+\frac{4}{1000}$
$(5\times10^{0})+(7\times10^{-1})+(2\times10^{-2})+\cdots$	3654.004
$6+\frac{7}{1000}$ 1 APPLICANT, 3 VACANCIES	$4+\frac{13}{1000}$ 2 APPLICANTS, 3 VACANCIES

YOU START FILLING SHELVES AT THE BACK.

6.007	4.013

ONLY AS AN EXERCISE, OTHERWISE SILLY.

DECIMAL TO FRACTION

$5.14=5\frac{7}{50}$	$3.03=3\frac{3}{100}$

RECURRING TO RATIONAL

$10\times.\dot{6}=6.\dot{6}$	$100\times.1\dot{8}=18.\dot{8}$	$100\times.\dot{1}\dot{8}=18.\dot{1}\dot{8}$
$-\ 1\times.\dot{6}=\ .\dot{6}$	$-\ 10\times.1\dot{8}=1.\dot{8}$	$-\ 1\times.\dot{1}\dot{8}=\ .\dot{1}\dot{8}$
$9\times.\dot{6}=6$	$90\times.1\dot{8}=17$	$99\times.\dot{1}\dot{8}=18$
$.\dot{6}=\frac{2}{3}$	$.1\dot{8}=\frac{17}{90}$	$.\dot{1}\dot{8}=\frac{2}{11}$

DECIMALS

FROM FRACTION TO DECIMAL

DIVIDE ($\div$ IS LIKE A VACANT FRACTION

	CALCULATOR	SHORT DIVISION
TERMINATING MEANS ENDING. (BUS TERMINAL, 1ST, 2ND, TERM) COMPARE	$\frac{3}{4} = 3 \div 4$ $.75$	$.75$ $3.00 \div 4$
TERMINATING	$\frac{3}{5} = 3 \div 5$ $.6$	$.6$ $3.0 \div 5$
TERMINATING	$\frac{1}{8} = 1 \div 8$ $.125$	$.125$ $1.000 \div 8$
RECURRING RE-OCCURRING, REPEATING	$\frac{1}{3} = 1 \div 3$ $.\dot{3}$	$.333\ldots$ $1.000 \div 3$
RECURRING	$\frac{2}{11} = 2 \div 11$ $.\dot{1}\dot{8}$	$.1818\ldots$ $2.0000 \div 11$
RECURRING	$\frac{7}{13} = 7 \div 13 = .\dot{5}3846\dot{1}$ DOT ON 1ST & LAST	
NEITHER	π PI $3.14159\,2654\ldots$	

PLACE VALUE IN DECIMALS

IS WHAT YOU HEAR, NOT IN WHAT COLUMN IT IS!
PULLING NUMBERS APART IS A USELESS EXERCISE!

IT'S ONLY PRESENTED HERE TO DEMONSTRATE HOW EFFICIENT THE DECIMAL SYSTEM IS, AND HOW TO READ NUMBERS.

SIMPLICITY PLUS	256,347 MEANS
THE PLACE VALUE OF 2 IS	200,000 200 THOUSAND
THE PLACE VALUE OF 5 IS	50,000 50 THOUSAND
THE PLACE VALUE OF 6 IS	6,000 6 THOUSAND
THE PLACE VALUE OF 3 IS	300 3 HUNDRED
THE PLACE VALUE OF 4 IS	40 FORTY (4X10)
PLACE VALUE IS FACE VALUE	7

TWO HUNDRED AND FIFTY-SIX THOUSAND, THREE HUNDRED AND FORTY-SEVEN.

DECIMALS: SHORT HAND WITH INDICES

$10^{\overset{\text{NEVER WRITTEN}}{1}}$ = A 1 WITH 1 NOUGHT: 10

$10^{\overset{\text{INDEX}}{2}}$ = A 1 WITH 2 NOUGHTS: 100

10^5 = A 1 WITH 5 NOUGHTS: 100 000

987,654,321 WRITTEN AS

$(9\times10^8)+(8\times10^7)+(7\times10^6)$
$+(6\times10^5)+(5\times10^4)+(4\times10^3)$
$+(3\times10^2)+(2\times10^1)+(1\times10^0)$

PRONOUNCED AS

NINE HUNDRED AND EIGHTY-SEVEN MILLION

SIX HUNDRED AND FIFTY-FOUR THOUSAND

THREE HUNDRED AND TWENTY-ONE

(ONE INDEX - TWO INDICES)

10^0 = A 1 WITHOUT NOUGHTS

$10^{-1} = .1 = \frac{1}{10}$ $\qquad$ $10^{-2} = .01 = \frac{1}{100}$

DECIMALS

MULTIPLICATION BY 10, 100, 1000, ...

ONE BILLION | HERE $10^{12} = 1000\ 000\ 000\ 000$ | U.S. $10^{9} = 1000\ 000\ 000$

DIGITS DO NOT CHANGE

ONLY THE DECIMAL POINT MOVES →

OBVIOUSLY

THE EYES COUNT. NO SILLY HOP, SCOTCH & JUMPS!

.000234	X 10 .00234	X 100 .0234	X 1000 .234
2.56	X 10 25.6	X 100 256	X 1000 2560

NO EXPLANATIONS — INTERESTED EYES WILL DISCOVER; THE BRAIN WILL REGISTER THE DISCOVERY.

DIVISION BY 10, 100, 1000, ...

ONLY THE DECIMAL POINT MOVES ←

OBVIOUSLY

234	÷ 10 23.4	÷ 100 2.34	÷ 10000 .0234

\$2.50 : \$3.25	ROUNDING OFF TO 15 NEAREST 10	267 N. 100	1500 N. 1000
10 : 13	20	300	2000
MORE THAN >	LESS THAN <	132.4715	132.4715
.21 > .19	.456 < .654	3 D.PLACES 132.472	2 D.PLACES 132.47
132.4715	2132	$8.4 \times 10^{13} \div 7 \times 10^{-2}$	$2.1 \times 10^{5} \div 7 \times 10^{-2}$
4 SIGNIFICANT FIGURES 132.5	1 SIGNIFICANT FIGURE 2000	1.2×10^{15}	$.3 \times 10^{7} =$ 3×10^{6}

DECIMAL POINT & DECIMAL FRACTION

THE **DECIMAL POINT** IS A **MARKER** ONLY;

IT SEPARATES THE WHOLE NUMBER PART AND THE FRACTION PART

IT HAS **NO VALUE**, SO DON'T INCLUDE IT IN COUNTING.

CONSEQUENTLY, **WHOLE NUMBERS** BY THEMSELVES ARE NOT FOLLOWED BY A DECIMAL POINT, EXCEPT IN CALCULATORS AS A MATTER OF CONSISTENCY.

$$125 \div 10 = 12.5$$

↑ **THINK** (AT FIRST) DECIMAL POINT NOT WRITTEN

EVENTUALLY, IT WILL BECOME A **ROUTINE** (WITHOUT THINKING)

SIMPLE FRACTION TO DECIMAL FRACTION

USE EQUIVALENT FRACTIONS (EQUAL VALUE) — ZIG-ZAG

$\frac{2}{5} \xrightarrow{\times 2} \frac{4}{10}$	THINK ONLY $\frac{3}{4} \overset{\times 25}{\underset{\times 25}{=}} \frac{75}{100}$	THINK ONLY $\frac{3}{20} \overset{\times 5}{\underset{\times 5}{=}} \frac{15}{100}$	$\frac{1}{8} = \frac{125}{1000}$

DO NOT EXPLAIN! **DECIMAL ⟷ DECIMAL FRACTION**

$.3$	$.51$	$.723$	$.9111$
$\frac{3}{10}$	$\frac{51}{100}$	$\frac{723}{1000}$	$\frac{9111}{10000}$

ALGORISMS FOR DECIMALS

ADDITION	SUBTRACTION
LINE UP DECIMAL POINTS	
1234.56789 + 111.11111 1345.67900 ALTHOUGH THE ZEROS HAVE NO VALUE, THEY SHOW THE **ACCURACY** OF THE ANSWER. 5 DECIMAL PLACES.	1234.56789 -1111.11111 123.45678 AWARENESS **OF COURSE**, WE DON'T WRITE A NOUGHT IN FRONT; IT'S NOT AN AREA CODE.
1234.56789 1 2 3 4 5 X .003 6 7 8 **IGNORE D. POINTS;** INSERT AFTERWARDS. HERE: TOTAL OF 8 PLACES. COUNTING FROM R TO L 3.70370367 8 7 6 5 4 3 2 1 EYES COUNT NO SILLY JUMPS PRACTISE SEEING, NOT JUMPING!	1234.5678 ÷ .002 **REWRITE** MOVE **BOTH** D.P.s 3 PLACES → 617283.9 1234567.8 ÷ 2 INSERT D.P. WHEN YOU COME TO IT.

DECIMALS BY CALCULATOR

$384.62 P.W.	$1685.28 P.M.	1. 1Kg @ $4.70	3. 750 g @ $3.60
$778.24 F.N.	$20152 P.A.	2. 300 g @ $1.38	4. $2\frac{1}{2}$kg @ $12.25

COMPARING INCOME

SINCE THE NUMBER OF DAYS IN A (CALENDAR) MONTH VARIES, CONVERT TO 1 YEAR.

384.62 X 52 ≑ 20.000

≑ → 778.24 X 26 ≑ 20.234

APPROX. 1685.28 X 12 ≑ 20213

COMPARING PRICES

BARK RECIPE — IF YOU WANT PRICE PER GRAM DO MONEY ÷ WEIGHT

IGNORE DECIMAL POINT

.47 .46 .48 .49

↑ CHEAPEST

HOW MANY $2.69 BOOKS FOR $500

YOU THINK: HOW MANY 2.69 IN 500, BUT YOU DO

500 ÷ 2.69 = 185

PRACTICAL ANSWER!

HOW MANY ZIPPERS TO SEW IN @ 90 CENTS EACH TO EARN $250

250 ÷ .9 = 278

FINISH THE LAST ONE!

$$\frac{3}{5} = \frac{16}{x} \qquad\qquad \frac{21}{46} = \frac{x}{111}$$

RECOGNISE EQUIVALENT FRACTIONS

ANOTHER BARK RECIPE: CROSS MULTIPLY SHORT CUT

TIMES DIVIDE

$\frac{3}{5} = \frac{16}{x}$ PUT 80 ÷ 3 OF COURSE

x ≑ 26.7

TIMES DIVIDE

$\frac{21}{46} = \frac{x}{111}$

x ≑ 50.7

MONEY DECIMAL SYSTEM

EUROPE €	AUSTRALIA $
COINS: 5-10-20-50 CENTS	COINS: 5-10-20-50 CENTS
1 & 2 EURO	1&2 DOLLAR (THALER, DAALDER)
NOTES: 5-10-20-50 EURO	NOTES: 5-10-20-50 DOLLAR

ROUNDING OFF

KOMMA	POINT
2,61 & 2,62 → 2,60	2.63 & 2.64 → 2.65

Europe

PAY € 50

COST € 32,73

BECOMES 32,75

CHANGE: SHOPKEEPERS WAY

5 CENTS	32,80
20 CENTS	33
€ 2	35
€ 5	40
€ 10	50
€ 17,25	

SOME COUNTRIES STILL USE 1 & 2 CENT COINS: CHANGE WOULD BE € 17,27

Australia

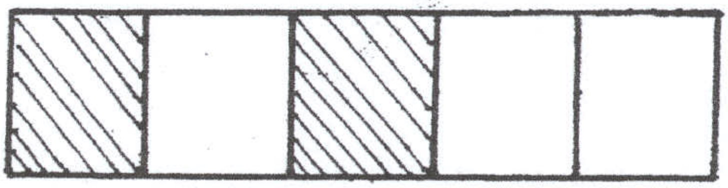
PAY $ 20

COST $ 11.35

CHANGE: THEY SAY

5 CENTS	11.40
10 CENTS	11.50
50 CENTS	12
3x $1 OR 2+1	15
$5	20
$ 8.65	

FRACTIONS



SHADED	2 OUT OF 5
WRITE	IN THAT ORDER $\frac{2}{5}$
SAY	TWO FIFTHS

SPECIAL NAMES		
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$
HALF	ONE (a) QUARTER	THREE QUARTERS

$1 - \frac{2}{5}$ THE NAME

UNDERSTANDING

ONCE ONLY

5 FIFTHS = ONE

− 2 FIFTHS

3 FIFTHS

ROUTINE FROM NOW ON

TAKE 2 FROM 5, KEEP THE NAME

FRACTION IN ACTION

NUMERATORS (COUNTERS)

								FULL HOUSE
$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9}$
								ONE

FRENCH **DENOMINATORS**

GIVING THE NAME OF THE FRACTIONS

			MIXED		MIXED
$1-\frac{3}{7}$	$\frac{4}{7}$	$4-\frac{3}{7}$	$3\frac{4}{7}$	$1+\frac{3}{7}$	$1\frac{3}{7}$
$1-\frac{4}{9}$	$\frac{5}{9}$	$2-\frac{4}{9}$	$1\frac{5}{9}$	$1+\frac{4}{9}$	$1\frac{4}{9}$
$1-\frac{1}{6}$	$\frac{5}{6}$	$3-\frac{1}{6}$	$2\frac{5}{6}$	$1+\frac{1}{6}$	$1\frac{1}{6}$
$1-\frac{5}{8}$		$5-\frac{5}{8}$	—	$1+\frac{5}{8}$	$1\frac{5}{8}$
$1-\frac{10}{11}$		$7-\frac{10}{11}$	—	$1+\frac{10}{11}$	$1\frac{10}{11}$
$1-\frac{4}{13}$		$6-\frac{4}{13}$	—	$1+\frac{4}{13}$	$1\frac{4}{13}$

FRACTIONS

PROPER	IMPROPER	MIXED
$\frac{\text{SMALLER}}{\text{BIGGER}}$	$\frac{\text{BIGGER}}{\text{SMALLER}}$	WHOLE $\frac{\text{SMALLER}}{\text{BIGGER}}$
3 QUARTERS AS YOU SAY IT $\frac{3}{4}$ FIRST ↓ SECOND THIRD	4 THIRDS DO NOT START WITH 4 $\frac{4}{3}$	ONE AND 1 THIRD $1\frac{1}{3}$

YOU START LAYING CONCRETE IN THE FAR CORNER, NOT AT THE DOOR!

FROM DICTATION

SURPRISE SECTION. BEWARE!

ONLY BY MAKING YOUR OWN SUMS,
DO YOU CREATE MULTI-LEVEL MATHS

FROM HERE ON ROUTINE

IF YOU EXPLAIN TOO MUCH, YOU EXPLAIN NOTHING!

NOTE: $\frac{7}{1}$ IS A RATIONAL NOT A FRACTION

WRITE SIMPLIFIED ONES.

$\frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$

TWO AND THREE FIFTHS

$2\frac{3}{5} \neq \frac{23}{5}$! (IS NOT)

FRACTIONS — THE BARK RECIPES

FROM IMPROPER TO MIXED

9 FIFTHS

$$\frac{9}{5} = 1\frac{4}{5}$$

DENOMINATOR

ONE AND 4 FIFTHS

EYES

BRAIN: HOW MANY 5s IN 9? 1, REMAINDER 4

(YELL IT!) AND THE NAME STAYS THE SAME

THIS SHOWS HOW PEDANTIC IT IS TO SAY.
"4 PLUS 5 EQUALS 9"
IT STOPS FLUENCY!

FROM MIXED TO IMPROPER

NOTE: IN THE RECIPE ITSELF TIMES PLUS!

PLUS

$$3\frac{2}{7} = \frac{23}{7}$$

TIMES

BUT DONE MENTALLY AS 21'n 2

AND THE NAME STAYS THE SAME.
WHEN YOU SAY IT, YOU MUST HEAR IT!

3'n 2 SEVENTHS = 23 SEVENTHS

WARNING: NEVER EXPLAIN FRACTIONS; THEY HAVE TO BE LEARNT AS A ROUTINE. YEAR 12 STUDENTS WILL REAP THE BENEFITS. 5-14 IS THE PERIOD OF PROGRAMMING.

MIXED ←TO→ IMPROPER

PLUS $1\frac{3}{5}$ TIMES	$\frac{8}{5}$	$\frac{13}{3}$ →	$4\frac{1}{3}$
$2\frac{1}{6}$	MENTALS —	$\frac{15}{4}$	
$3\frac{5}{7}$	—	$\frac{9}{2}$	
$4\frac{2}{3}$	—	$\frac{7}{2}$	
$5\frac{5}{8}$	—	$\frac{23}{5}$	
$6\frac{2}{3}$	—	$\frac{31}{6}$	MENTALLY YOU SAY HOW MANY 6s IN 31? 5 R1, WRITE AS $5\frac{1}{6}$
$7\frac{3}{4}$	—	$\frac{43}{7}$	CALCULATOR YOU DO 43 a b/c 7 =

EQUAL VALUE

EQUIVALENT FRACTIONS

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} \ldots$$

MENTALLY SIMPLIFYING FRACTIONS

DO NOT CROSS OUT NUMBERS

$\frac{10}{15} = \frac{2}{3}$ | $\frac{21}{24} = \frac{7}{8}$ | $\frac{15}{35} = \frac{3}{7}$ | $\frac{14}{70} = \frac{1}{5}$ (THINK 2/10)

TYPE 1

ARRANGE IN SMALL → BIG / DOWN

ASCENDING OR DESCENDING ORDER

ORIGINAL ORDER	$\frac{3}{4}$ ×3	$\frac{7}{12}$	$\frac{1}{2}$ ×6	$\frac{5}{6}$ ×2	$\frac{2}{3}$ ×4
EQUIVALENT FRACTIONS SAME DENOMINATOR	$\frac{9}{12}$	$\frac{7}{12}$	$\frac{6}{12}$	$\frac{10}{12}$	$\frac{8}{12}$
ASCENDING ORDER	$\frac{1}{2}$	$\frac{7}{12}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$

TYPE 2

DESCENDING

$\frac{3}{5}$ $\frac{4}{7}$ $\frac{5}{6}$

CALCULATOR: $3 \div 5 = .60$ $4 \div 7 \doteq .57$ $5 \div 6 \doteq .83$

IN DESCENDING ORDER: $5/6$ $3/5$ $4/7$

FRACTIONS	THE 4 NON-SURGICAL OPERATIONS

TYPE 1	SAME NAME (DE$\overset{\text{FRENCH}}{\text{NOM}}$INATOR)

$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ IN WORDS: 3 SEVENTHS + 2 SEVENTHS IS 5 SEVENTHS	$\frac{8}{9} - \frac{3}{9} = \frac{5}{9}$ IN WORDS: 8 NINTHS − 3 NINTHS IS 5 NINTHS

TYPE 2	RELATED NAMES: NUMBER & MULTIPLE

$\frac{4^{\times 2}}{5_{\times 2}} + \frac{1}{10} = \frac{9}{10}$	$\frac{5}{24} - \frac{1^{\times 4}}{6_{\times 4}} = \frac{1}{24}$

TYPE 3	DIFFERENT NAMES; NOT RELATED

$\frac{2}{3} + \frac{1}{5} = \frac{13}{15}$ (arrows: 1, 2, 3) RECIPE: TIMES. TIMES. TIMES EYES SEE: $10 + 3 = 13$ $3 \times 5 = 15$	$\frac{2}{3} - \frac{1}{5} = \frac{7}{15}$ RECIPE: TIMES. TIMES. TIMES EYES SEE: $10 - 3 = 7$ $3 \times 5 = 15$
$\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$ RECIPE: TIMES TIMES ACROSS	$\frac{2}{3} \div \frac{1}{5} = 3\frac{1}{3}$ (arrows: 1, 2) SHORTEST RECIPE! TIMES. TIMES SEE $\frac{10}{3}$, BUT WRITE → $3\frac{1}{3}$

FRACTIONS: AN EXERCISE IN SEEING & THINKING

MULTIPLICATION SHORTCUTS

TIMES TIMES ACROSS, BUT

CANCEL FIRST RATHER THAN FORGETTING TO DO IT AFTERWARDS.

NOTE! IT IS SILLY TO CROSS OUT WHAT YOU'VE SEEN!

$\frac{5}{11} \times \frac{2}{15} = \frac{2}{33}$	$\frac{6}{13} \times \frac{5}{9} = \frac{10}{39}$
$\frac{4}{15} \times \frac{3}{8} = \frac{1}{10}$	$\frac{7}{20} \times \frac{8}{21} = \frac{2}{15}$

RECIPROCALS	AVERAGE or MEAN $\bar{x}$ (4, 7 & 11)
TURNING FRACTIONS UPSIDE DOWN	ADD & DIVIDE BY NUMBER OF SCORES
$\frac{6}{7} \rightarrow \frac{7}{6} = 1\frac{1}{6}$	$\frac{4+7+11}{3}$ NUMBERS
$3\frac{1}{2} = \frac{7}{2} \rightarrow \frac{2}{7}$	
$7 = \frac{7}{1} \rightarrow \frac{1}{7}$	$7\frac{1}{3}$

OPERATIONS WITH MIXED FRACTIONS

ADDITION

NO PROBLEM

ADD INTEGERS & TIMES.TIMES.TIMES

$$1\frac{3}{5} + 3\frac{1}{6} = 4\frac{23}{30}$$

SUBTRACTION	MULTIPLICATION	DIVISION
CONVERT TO IMPROPER FIRST		
TO AVOID COMPLICATIONS	NECESSARY	NECESSARY
$4\frac{1}{6} - 1\frac{3}{5} =$	$3\frac{2}{3} \times 2\frac{1}{5} =$	$3\frac{2}{3} \div 2\frac{1}{5} =$
$\frac{25}{6} - \frac{8}{5} = \frac{77}{30}$	$\frac{11}{3} \times \frac{11}{5} = \frac{121}{15}$	$\frac{11}{3} \div \frac{11}{5} = \frac{5}{3}$
$2\frac{17}{30}$	$8\frac{1}{15}$	$1\frac{2}{3}$

FRACTIONS ALL SORTS

$\frac{1}{2}$ OF $\frac{3}{5}$ OF 40 12	$\frac{1}{4}$ OF 60 IS $\frac{1}{3}$ OF... 45
$\frac{1}{3}$ OF YEAR 10 PLAYS TENNIS $\frac{1}{2}$ SOCCER, 18 NO SPORT HOW MANY IN YEAR 10?	YOU TRAVELLED $\frac{3}{5}$ OF THE 450 km TRIP. HOW FAR TO GO?
$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ SPORT $\therefore \frac{1}{6}$ YR 10 = 18 YR 10: TOTAL 108 STUDENTS	$\frac{2}{5}$ OF 450 = 180 km

A WHOLE NUMBER X A FRACTION

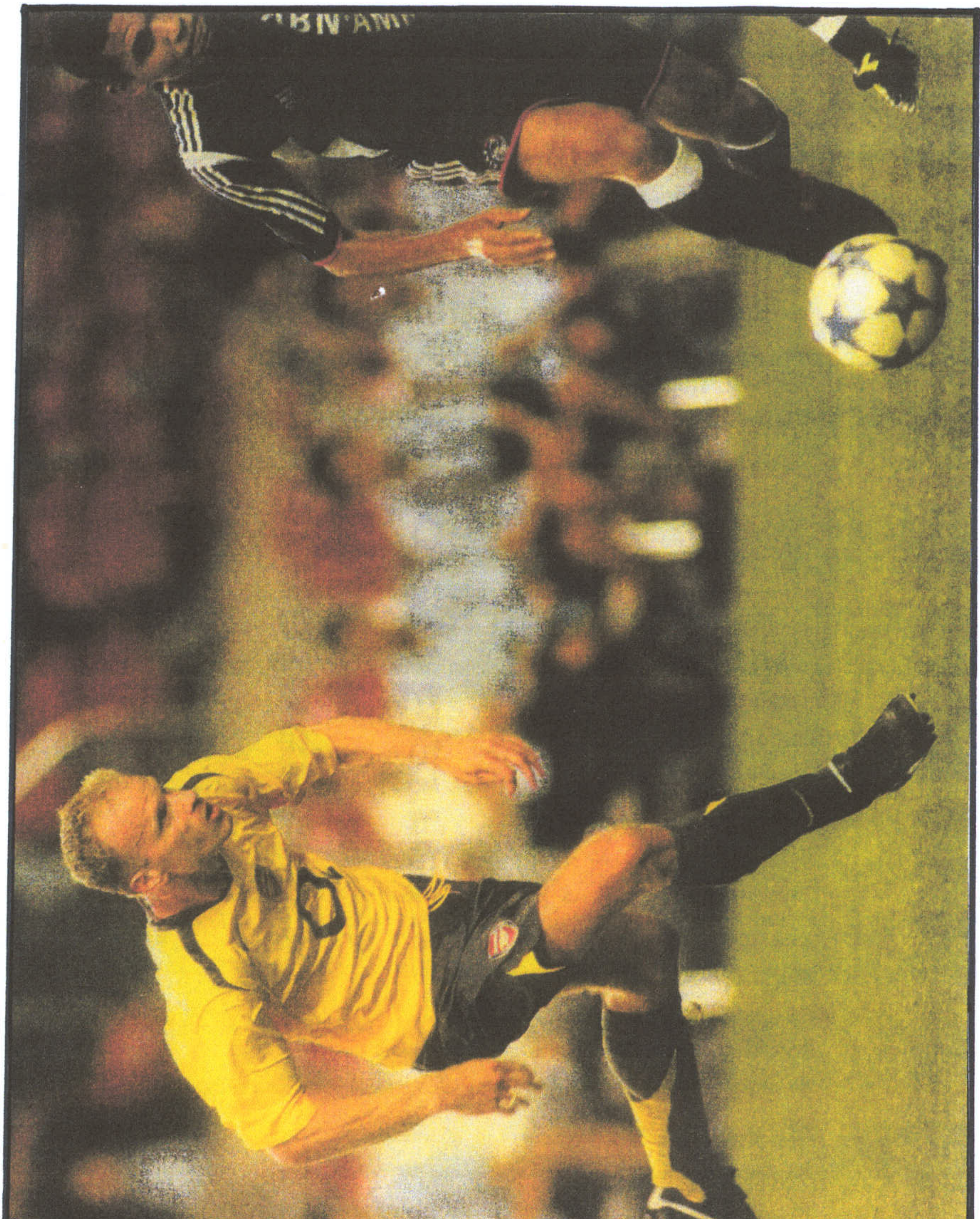
2 X 3 APPLES = 6 APPLES

LIKEWISE

2 X 3 SEVENTHS = 6 SEVENTHS

WRITTEN AS $2 \times \frac{3}{7} = \frac{6}{7}$

$4 \times \frac{1}{7} = \frac{4}{7}$	$4 \times \frac{2}{9} = \frac{8}{9}$
$6 \times \frac{1}{3} = 2$!	$3 \times \frac{2}{17} = \frac{6}{17}$
$2 \times \frac{3}{16} = \frac{3}{8}$! CANCEL MENTALLY	$5 \times \frac{5}{12} = 2\frac{1}{12}$ THINK $\frac{25}{12}$
$7 \times \frac{3}{14} = 1\frac{1}{2}$ CANCEL FIRST	$4 \times \frac{5}{12} = 1\frac{2}{3}$



ATTENTION & CONCENTRATION

A FRACTION OF A NUMBER

ROUTINE (NO EXPLANATIONS!) EYES SEE BRAIN REACTS

$\frac{1}{4}$ OF 16 **MEANS**

Now we use HOW MANY 4s IN 16

IF $\frac{1}{4}$ OF 16 = 4, $\frac{3}{4}$ OF 16 = 12

$\frac{1}{4}$ OF 8 = 2	$\frac{3}{4}$ OF 8 = 6
$\frac{1}{5}$ OF 20 = 4	$\frac{3}{5}$ OF 20 =
$\frac{1}{6}$ OF 72 =	$\frac{5}{6}$ OF 72 =
$\frac{1}{7}$ OF 14 =	$\frac{4}{7}$ OF 14 =
EYES & BRAIN $\frac{5}{8}$ OF 16 = 10 8 INTO 16 = 2, TIMES 5	$\frac{7}{9}$ OF 18 =
$\frac{3}{7}$ OF 28 =	$\frac{2}{3}$ OF 24 =

FRACTONS ALL SORTS

YOU TRAVELLED
450 km WHICH IS $\frac{3}{5}$ TRIP
HOW FAR TO GO?

IF $3/5$ IS 450
$2/5$ IS 300
$\left(\frac{3}{2} \; \frac{450}{x = 300}\right)$

$8 PER HOUR.
40 NORMAL, 4 DOUBLE AND
3 HOURS "TIME AND A HALF"

$(40 \times 8) + (4 \times 16) + (3 \times 12) =$
$420

$\frac{5}{8}$ LITRE

$\frac{5}{8} \times 1000$

625 ml

$\frac{3}{4}$ OF $1\frac{7}{9}$

$\frac{3}{4} \times \frac{16}{9}$

CANCEL FIRST! $1\frac{1}{3}$

$\frac{1}{8} + x = \frac{3}{4}$

$x = \frac{3}{4} - \frac{1}{8} = \frac{5}{8}$

$40 \div \left(1 - \frac{3}{7}\right)$

$40 \div \frac{4}{7}$ CHANGE TO
$40 \times \frac{7}{4} = 70$
TIMES RECIPROCAL

IF $\frac{2}{7}$ OF A NUMBER IS 12, FIND IT

POTATO SUM
DONE MENTALLY
$\frac{2}{7} \; \frac{12}{x = 42}$

$\frac{2}{3}$ OF $183

MENTALLY
$\frac{2}{3} \times 183$
$122

RECIPROCAL OF $2\frac{1}{5}$

$5/11$

AVERAGE OF
5, 6, 7 & 3

$5\frac{1}{4}$

$1\frac{1}{3} \div 2\frac{3}{5}$

COMPARE

$\frac{4}{3} \div \frac{13}{5} = \frac{20}{39}$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

$\frac{3}{4} + \frac{1}{3} = 1\frac{1}{12}$

$\frac{1}{8} + \frac{5}{6} + \frac{1}{2} + \frac{1}{3}$

$\frac{5}{8} + 1\frac{1}{6} = 1\frac{19}{24}$

$\frac{1}{2} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{6}$

ORDER OF OPERATION

$\frac{1}{12} + \frac{2}{3} = \frac{3}{4}$

INTEGERS

FRACTIONS IN WHOLE NUMBERS

	THINK	SAY
HOW MANY HALVES IN 1	2X1	2
HOW MANY THIRDS IN 2	3X2	
HOW MANY QUARTERS IN 3	4X3	
HOW MANY $\frac{3}{4}$s IN 3	12÷3	4
HOW MANY FIFTHS IN 4		
HOW MANY 2FIFTHS IN 4		
HOW MANY SIXTHS IN 5		
HOW MANY 5SIXTHS IN 5	SEE	6
HOW MANY SEVENTHS IN 2		
HOW MANY $\frac{2}{7}$s IN 2	SEE	7
HOW MANY $\frac{4}{7}$s IN 2	14÷4	$3\frac{1}{2}$
HOW MANY EIGHTHS IN 7		
HOW MANY 7/8s IN 7	SEE	
HOW MANY NINTHS IN 8		
HOW MANY 4/9s IN 8	72÷4	
HOW MANY 8/9 IN 8	SEE	

WHAT FRACTION IS $\frac{\text{THIS}}{\text{OF THAT}}$

$$\frac{9}{\text{OF } 126} = \frac{1}{14}$$

$$\frac{24 \text{ cm}}{\text{OF } 1 \text{ m}} = \frac{24}{100} = \frac{6}{25}$$

$$\frac{4 \text{ WEEKS}}{\text{OF } 1 \text{ YEAR}} = \frac{4}{52} = \frac{1}{13}$$

5 SCHOOLDAYS + 2 WEEKS: 5 LETTERS 2 Es

$$\frac{450 \text{ g}}{\text{OF } 1 \text{ kg}} = \frac{45}{100} = \frac{9}{20}$$

CALCULATOR 45 $a^{b}/_{c}$ 100 =

$$\frac{3 \text{ HOURS}}{\text{OF } 1 \text{ DAY}} = \frac{3}{24} = \frac{1}{8}$$

$$\frac{325 \text{ ml}}{\text{OF } 2 \text{ L}} = \frac{325}{2000} = \frac{13}{80}$$

FRACTIONS | INDICES AND ROOTS

SQUARE ROOT, THE RADICAL SIGN WITHOUT 2
SQUARE, x^2 WITH 2

COMPARE	$\sqrt[3]{x}$	$\sqrt[4]{x}$	$\sqrt[5]{x}$	$\sqrt[5]{32} = 2$	$\sqrt[6]{64} = 2$
	x^3	x^4	x^5	$2^5 = 32$	$2^6 = 64$

$\sqrt{\frac{4}{81}} = \pm\frac{2}{9}$	$\sqrt{\frac{121}{64}} = \pm 1\frac{3}{8}$	$\sqrt{\frac{1}{100}} = \pm\frac{1}{10}$	$\sqrt[3]{\frac{27}{125}} = \frac{3}{5}$

$10\,000^{\frac{1}{2}}$	$1000\,000^{-\frac{1}{2}}$
ANOTHER WAY OF WRITING $\sqrt{10\,000} = 100$ (4 NOUGHTS, 2 NOUGHTS)	MEANS $\frac{1}{\sqrt{1000\,000}} = \frac{1}{1000}$

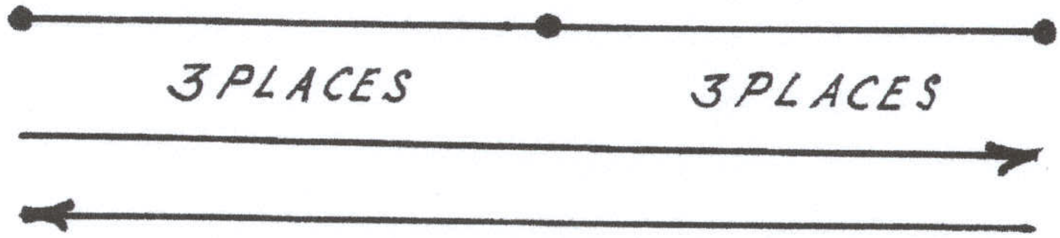
$9^{\frac{1}{2}}$	$\sqrt{9}$	± 3	$9^{-\frac{1}{2}}$	$\frac{1}{\sqrt{9}}$	MENTALLY $\pm\frac{1}{3}$
$27^{\frac{1}{3}}$	$\sqrt[3]{27}$	3	CALCULATOR $9^{-\frac{1}{2}}$ 9 x^y 1 a^b/c 2 +/- = $(\frac{1}{3} = 1 \div 3 = .\dot{3})$ $\pm .\dot{3} = \frac{1}{3}$		
$27^{\frac{2}{3}}$	$\sqrt[3]{27^2}$	CUBE ROOT IS 3, SQUARE IT 9	$27^{-\frac{2}{3}}$	MENTALLY $\frac{1}{9}$	$= 1 \div 9$
			CASIO 27 x^y 2 a^b/c 3 +/- = $.\dot{1}$		
$32^{\frac{1}{5}}$ (32 x^y 1 a^b/c 5)	$\sqrt[5]{32}$	2	$32^{-\frac{2}{5}}$	$\frac{1}{\sqrt[5]{32}^2}$	FIFTH ROOT OF 32 IS 2, SQUARE IT; ANSWER $\frac{1}{4}$

MEASUREMENT: CONVERSIONS

ONE SYSTEM

LIKE THE ROAD RULES: APPLIED, NOT EXPLAINED!

KILO-	HECTO-	DECA-		DECI-	CENTI	MILLI-
km	100	10	m	1/10	1/100 cm	1/1000 mm
kg			g			mg
kl			l			ml



1 km = 1000 m 50 m = 5 cm	1 m = 100 cm 500 cm = 5 m	1 cm = 10 mm 700 m = .7 km

$$20\ m/_{sec} = .02 \times 3600 = 72\ km/_h$$

THINK 2X36!

$$1\ km/_h = 1000\ m \text{ PER } 3600 \text{ SECONDS}$$

$$= \frac{10}{36} = \frac{5}{18}\ m/_{sec.}$$

MEASUREMENT

1 KNOT = 1 NAUTICAL MILE (PER HOUR) 1 MINUTE OF (EQUATOR) A GREATCIRCLE MEASURED IN EQUADOR 1852 m	1 TONNE 1000 kg 1 MEGATONNE 1 Mt = 10^6 t	1 micro gram 1 μg = 10^{-6} g, 1 millionth of a gram	1 micron is 10^{-6} m, 1 millionth of a metre.

HOW TO REMEMBER CALCULATOR

8
5
1 2

AREA

1 km^2	1000 X 1000 = 10^6 m^2 100 hectare	FROM 10:12 am TO 3:42 pm	10:42 am 3:12 p.m	FROM 4° BELOW TO 10° ABOVE
1 hectare	1 $Hectometre^2$ 100 X 100 = 10 000 m^2	15:42 - 10:12 5 HRS. 30 MIN.	14:72 - 10:42 4 HRS. 30 MIN	14°
1 m^2	100 X 100 = 10 000 cm^2	VOLUME 1 m^3	100 X 100 X 100 = 10^6 cm^3 1000 litres 1000 kg WATER	
		ONE	mega litre = 10^6 litres.	
1 cm^2	10 X 10 = 100 mm^2	1 cm^3	10 X 10 X 10 = 1000 mm^3	

$\frac{1}{2}$ TONNE OIL SPILL 1 kg = 1.3 litre DEPTH 2 mm	SURFACE AREA CUBE 337.5 cm^2 6.8 g PER cm^3	$C = \frac{5}{9}(F - 32)$ FAHRENHEIT
V = 500 X 1.3 = 650 l WHICH IS .65 m^3 (1 m^3 = 1000 l) DIVIDE BY .002 AREA COVERED 325 000 m^2	EDGE x cm 6 FACES = 337.5 cm^2 1 FACE = 56.25 cm^2 = x^2 SQ. ROOT, x = 7.5 cm VOLUME = 7.5^3 cm^3 = 421.875 cm^3 x 6.8 WEIGHT = 2.86875 Kg.	TEMPERATURE CONVERSION 62° F = 150 ÷ 9 ≑ 17° C IF CELSIUS = 20° F = 20 X 9 ÷ 5 + 32 = 69°

ORDER OF OPERATION

1.	BRACKETS	[{ (⟶
2.	TIMES & DIVISION LEFT → RIGHT	NEGATIVE = NOT
3.	POSITIVES & NEGATIVES	SEE WHO IS WINNING

$8+9-13$	4	$13-22$	-9
$13+15\div3$	18	$20-3\times4$	8
$6+10\times(4-2)$	26	$25\times1.37\times4$	A.L.M 137
$(81\times2.61)+(19\times2.61)$	261	UNDERSTANDING ! $24\div(-8)$ (− = NOT; COMBINE 2 SIGNS; NOT POS. IS)	-3

MENTAL ARITHMETIC MEANS EFFICIENCY

CALCULATOR		LEFT → RIGHT	
$60\div5-4\times3$	0	$(14\times3.7)-(4\times3.7)$	37
$8+7\times13$	99	$72\div6\div3$	4
$(5+7\times4-3)\div10+3$	6	$[2(4+5)-2^3]^2$	100
$(-3)\times5\times(-2)$ (NOT … NEG IS POS.)	30	$8\times(-6)\div(-12)$ (NOT … NEG IS POS)	4

PERCENTAGE

PER 100. SYMBOL %

IT STILL IS A 1 & 2 NOUGHTS

NOTE:

FRACTIONS & % ARE WRITTEN IN THE SAME ORDER.

AS YOU SAY IT. DO NOT START WITH — OR / A SILLY HABIT!

THE WHOLE IDEA IS LOST.

3% HAS REPLACED THE CLUMSY DECIMAL FRACTION $\frac{3}{100}$

IN CALCULATIONS, MENTALLY CONVERT TO DECIMALS; IT'S MORE EFFICIENT THAN USING THE % BUTTON!

ONLY THE DP (DECIMAL POINT) MOVES

FROM DECIMAL TWO PERCENT FROM

PLACES

3% = .03 THINK	14% = .14 THINK	12.5% = .125

MENTAL APPLICATIONS OF THE

FAMOUS DOZEN

PERCENT	OF	NUMBER	THINK	ANSWER
100	OF	24	1	24
75	OF	16	3/4	12
66 2/3	OF	27	2/3	18
50	OF	30	1/2	15
33 1/3	OF	30	1/3	10
25	OF	24	1/4	6
12 1/2	OF	96	1/8	12
10	OF	70	1/10	7
5	OF	60	1/20	3
4	OF	75	1/25	3
2	OF	150	1/50	3
1	OF	400	1/100	4

3 4/5 = 380%	2 3/4 = 275%	11/20 = 55%

IN CALCULATIONS, THE FAMOUS DOZEN ARE MENTALLY CONVERTED TO FRACTIONS WHEN DEALING WITH SUITABLE NUMBERS. (1)
OTHERS ARE MENTALLY TURNED INTO DECIMALS, EVEN WHEN USING A CALCULATOR; ONE ROUTINE! (2)

—A PERCENTAGE OF MEANS X

1. PRICE \$20, 75% OFF. PAY $\frac{1}{4} \times 20 = \$5$

2. 16% BECOMES .16

4% BECOMES .04

7½% BECOMES .075 BECAUSE 7½ = 7.5

THINK

16 = 16. AS BY CALCULATOR

∴ 16% = .16 ← DECIMAL POINT

2 PLACES BACK (100 HAS 2 NOUGHTS!)

NOTE: 2 PLACES BACK EITHER IN THE % OR IN THE AMOUNT, WHICHEVER IS MORE EFFICIENT!

PERCENTAGE

EFFICIENCY SHOULD BE PART OF EDUCATION!

TYPE 1.	PRACTISING EYES TO SEE MORE
1. MENTALLY	6% OF 200 THINK 6X2, WRITE OR SAY 12
TYPE 2 MENTALLY	12% OF 7000 THINK 12X70, WRITE OR SAY 840
TYPE 3 MENTALLY	13% OF 60 THINK 1.3X6, WRITE OR SAY 7.8
TYPE 4 ALGORISM OR CALCULATOR	15% OF $24.30 DO .15 X 24.3 !
TYPE 5 ALGORISM OR CALCULATOR	24% OF 85 DO .24 X 85 STANDARD ROUTINE

2.	WHAT PERCENTAGE IS $\frac{\text{THIS}}{\text{OF TH.AT.}}$
A.	THINK 2 PLACES BACK MARK: $\frac{69}{\text{OUT OF } 120}$ BY CALCULATOR: DO $69 \div 1.2 = 57.5\%$
B.	$\frac{54}{\text{OF } 80}$ ALGORISM $\overset{67.5}{540 \div 8}$ BY CALCULATOR: DO $54 \div .8 = 67.5\%$
C.	$\frac{460\ g}{\text{OF } 2\ Kg}$ MENTALLY DO $46 \div 2 = 23\%$
D	$\frac{13 \text{ WEEKS}}{\text{OF } 1 \text{ YEAR}}$ 5 LETTERS 2 WEEKS MENTALLY DO $\frac{13}{52} = \frac{1}{4} = 25\%$ FAMOUS DOZEN

3. # PERCENTAGE

SYMBOL FOR 100

IF 20%

OF A NUMBER IS 30,

A. WHAT'S THE WHOLE NUMBER MEANS WHAT'S 100%

MENTALLY: 150

B. ## GENERAL ROUTINE !

USUALLY BY CALCULATOR

IF 14% OF x (OF A NUMBER) IS 98

→ READ

← DO

NUMBERS IN REVERSE

$x = 98 \div .14 = 700$

THINK 2 PLACES BACK

FROM PERCENTAGE TO DECIMAL

PERCENTAGE

	A WHOLE IS 100%	
4.	21% POPULATION IN SYNEY ∴ 79% ELSEWHERE	CATFOOD: 20% FISH, 75% CEREAL ∴ 5% OTHER INGREDIENTS
5.	$7\frac{1}{2}$% OF 1 metre = $7\frac{1}{2}$ centimetre.	13% OF $1 = 13 cents.
6.	+ INCREASE (SO IT BECOMES 120% = 1.2) 560 BY 20% ($\frac{1}{5}$) MENTALLY 672. ONE STEP CALCULATOR 560 X 1.2	− DECREASE 560 BY 25% ($\frac{1}{4}$) 2 STEPS. 420 ONE STEP CALCULATOR 560 X .75
7. A.	COST $60. SELL $80. PROFIT $20 — 33% ($\frac{1}{3}$) OF COST. 25% ($\frac{1}{4}$) OF SALES PRICE	
B.	COST $2680. PROFIT 22%. IN ONE GO! SALES PRICE 2680 X 1.22 = $3269.60 (PUT 0)	
C.	4 TYRES @ $78.40 EACH. DISCOUNT 15%. IN ONE GO! 4 X 78.4 X .85 = $266.56 (DO NOT USE 0)	

PERCENTAGE THE PROFESSIONAL WAY

8. COMMISSION ONCE ROYAL REAL ESTATE (IN THE PAST)

LAND \$82000: 3% FIRST 15000, 2% ON 30000, THEN 1.5%

3X150 + 2X300 + 1.5X370 = \$1605

9. RADIO \$245, 15% DEPOSIT + \$4.20 P.W.

FOR 1 YEAR. CALCULATE INTEREST CHARGED

INTEREST IS CHARGED ONLY ON .85X245 = 208.25

.15X245 + 52X4.2 − 245 = ...

DIVIDE BY 2.0825 ≑ 4.9% REMEMBER:

WHAT % IS THIS OF THAT? SLICE LOAF 2 PLACES BACK

10 GROSS INCOME \$520 P.W. TAX 28%

NET .72X520 = \$374.40

↑PUT YOURSELF!

11 CARPART \$90, 15% DISCOUNT, 10% G.S.T

.85X90 = ... X1.1 = \$84.15

12 COST INCLUDING 15% OFF IS \$80

SEE NO. 3 MARKED PRICE WAS 80 ÷ .85 = \$94.12

13 15% DISCOUNT OR \$20

SEE NO. 3 20 ÷ .15 = \$133.33 MARKED PRICE

14 36 BOYS, 12 MORE GIRLS. WHAT % OF TOTAL

SEE NO. 2 48 ÷ .84 ≑ 57%

15 BOYS (450) : GIRLS = 3 : 4 POTATO SUM $\frac{3}{4} = \frac{450}{x}$

NO. 2 600 GIRLS = 600 ÷ 10.5 = 57% BOYS 43%

PERCENTAGE THE PROFESSIONAL WAY

16. 120 STAFF. M:W = 5:3
FIND NEW RATIO AFTER 20% & 40% INCREASE

1. 5/8 × 120 = 75 MEN ∴ 45 WOMEN
2. 90 MEN & 63 WOMEN NEW RATIO 10:7

17. 2 NUMBERS. SUM 540. ONE IS 70% LARGER

SO WE HAVE 100% & 170%

∴ $x + 1.7x = 2.7x = 540$ ∴ $x = 200$ & $1.7x = 340$

18 DIAMOND INCREASED IN PRICE RATIO 20:29
NEW PRICE $376 POTATO SUM

IF 29 — 376 TIMES DIVIDE
THEN 20 — x = $259.31

19 $900 @ 24% p.a. FOR 6 YEARS

IT IS VERY STUPID TO WRITE DOWN FIRST WHAT YOU'RE GOING TO PUT IN THE CALCULATOR! CUSTOMERS ARE WAITING. PRESS 9X24X6 = $1296

INTEREST AT A FLAT RATE

20 $2800 @ 9.5% p.a. FOR 7 MONTHS

PRESS 28 × 9.5 × 7 ÷ 12 $155.17

21. $32 400 @ 1.76% p.m. FOR 14 MONTHS

PRESS 1.76 × 324 × 14 $7 983.36

22. BORROW $1600 @ 12% p.a. 4 YEARS

REPAY 1600 + 16 × 48 = $2368

23. PRINCIPAL $4550 INTEREST $2912 4 YRS

$728 p.a WHAT % IS 728 OF 4550?
SLICE LOAF 2 PLACES BACK 728 ÷ 45.5 = 16% FLAT

PERCENTAGE THE PROFESSIONAL WAY

24. LAND RATES BASED ON U.C.V.

(LAND ONLY) UNIMPROVED CAPITAL VALUE

VALUER GENERAL'S ASSESSMENT. (NOT MARKET VALUE!)

U.C.V. \$28000 1.24 CENTS IN THE DOLLAR A STUPID EXPRESSION FOR 1.24%	U.C.V. \$54000 RATES \$610.20
PRESS 1.24 × 280 ANSWER \$347.20 p.a.	SLICE LOAF 2 PLACES BACK 610.2 ÷ 540 = 1.13%

25 COMPOUND INTEREST. INTEREST ON PRINCIPAL & INTEREST

\$400 @ 12% p.a. 10 YEARS. NO FORMULA!

IT'S A VISUAL! PRESS $1.12^{10} \times 400 = \$1242.34$

$1.12\ x^y\ 10 \times 400$

26. DEPRECIATION \$7000 @ 15% p.a. 5 YRS

VALUE AFTER 5 YEARS $.85^5 \times 7000$

BUSINESS $.85\ x^y\ 5 \times 7000 = \$3106$

FOR TAX DEDUCTION PURPOSE IN STAGES

.85 ×× 7000 = 5950 = 5057.5 = 4298.88 = 3654.04 =

1 2 3 4 5

27. \$8000 INVESTMENT @ 12% p.a. (= 1% p.m) 4 YEARS

MONTHLY RESTS MEANS COMPOUNDED MONTHLY

ACCRUED VALUE $1.01^{48} \times 8000 = \$12\,898$

28. PERSONAL LOAN \$5000 @ 18% p.a. 4 & 5 YRS COMPARE

REPAY 5000 + 50 × 72 = . . .	REPAY 5000 + 50 × 90 = . . .
DIVIDE BY 48 = \$179.17 p.m.	DIVIDE BY 60 = \$158.33

FIRS LEARN ROUTINE	# PERCENTAGE		THE PREVIOUS PROCEDURES
64% AS A FRACTION	$8\frac{1}{3}$% AS A DECIMAL	.47 D → P (2)	.018
$\frac{16}{25}$	$.08\dot{3}$	47%	1.8%
13% D ← P (2)	1.25%	1.25 D → P (2)	2.13
.13	.012	125%	213%
6%	7.5% OF 1m	60% OF $1	INCREASE 200 BY 1.07%
.06	75 mm	60 CENTS	2 X 101.07
DECREASE 200 BY .93%	13% OF 400	1.2% OF 300	30% OF 50
2 X 99.07	52	3.6	15
WHAT % IS 34 OF 630	25% OF x IS 16	62% OF x IS 8	COST $340 32% PROFIT
34 ÷ 6.3	$x = 64$	8 ÷ .62	132 X 3.4
COST $520, SELL $700		SELL $610 LESS 15%	
PROFIT $180	180 ÷ 5.2 = 34.6%	SELL FOR	85 X 6.1 = $518.50

PERCENTTAGE

15% OFF OR $35	$320 @18% P.A. 3MONTHS	EARN $75 SPEND 45	1m LESS 4%
PRICE 35÷.15	INTEREST 4.5X3.2	LEFT 30÷.75	96 cm
DIFFERENCE: 45% OF 150 & 35% OF 180 (4.5)	INCOME $x SPEND $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ $=\frac{20+15+12+10}{60}$	$6\frac{1}{4}$% P.A. ON $1684, 40 DAYS INTEREST	INTEREST UP FROM $7\frac{1}{2}$% TO $8\frac{1}{4}$%. INCREASE $ 71.25
45x1.5−35x1.8	LEFT $\frac{1}{20}$=5%	16.84X6.25 ÷365X40	INCREASE 3/4% LOAN $9500 (71.25÷.0075)
DEPRECIATION ON $9540 1ST YEAR 32% THEN 25% NEW VALUE $3649	$64.70 − 10% THEN 15% OFF FOR CASH	764 + 17.5%	$167.44 INCLUDING 8% DISCOUNT
9540x.68=… x.75=… x.75= 3649 ∴ 3 YRS	.9X64.7=… PAY x.85=$49.50	1.175X764= 897.7	ORIGINAL PRICE 167.44÷.92= $182
RENT HOME UNIT COST $56000 COLLECT $85 P.W. COST $960 P.A.	FLAT RATE $ 2500 16 M @ $11\frac{1}{2}$% P.A.	DEPRECIATION 20% OF $8400 3 YEARS	FISH: 135 Kg FOR $360 75 Kg FILLET LEFT. 50% PROFIT
NET RETURN 85x52−960= $3460 DIVIDE BY 560 ÷ 6.2%	INTEREST 25x11.5÷12x16 $ 383.36	NEW VALUE $.8^3$x8400= $ 4301	SELL FOR 360x1.5÷75= $7.20 PER KILO

PERCENTAGE

LOAN \$4500 6 M.
@ 4% PER ANNUM

REPAY 4500 + 90 =
\$ 4590

LOAN \$ 6600, 5 M
INTEREST \$ 233.75

INTEREST FOR 1 YEAR:
233.75 ÷ 5 X 12 = ...
THINK, "WHAT % IS $\frac{\text{THIS}}{\text{OF THAT}}$."
DIVIDE BY 66 = 8.5% P.A.

PAY \$ 2295 INTEREST
IN 6 YEARS @ 8.5%

INTEREST 1 YEAR: 2295 ÷ 6 = ...
DIVIDE BY .085 = \$ 4500
LOAN

15% OFF
PAY \$ 54.40

MARKED PRICE
54.4 ÷ .85 = \$ 64

15% OFF OR \$ 10
IF
10 ÷ .15
MARKED PRICE \$66.67

78% OF x = 9282

x = 9282 ÷ .78 = 11900

78% OF x = 9282

46% OF x = 9282 ÷ 78 X 46

500 KG @ 2.24 PER KILO
PROFIT \$420 AS A %

420 ÷ (5 X 2.24 = 37.5% ON COST

\$ 620 CASH OR 15%
DEPOSIT + \$26.10 P.M. 2 Y
CALCULATE INTEREST CHARGED

DEPOSIT \$93 ∴ INTEREST ONLY ON 527
PAID 24 X 26.1 = \$626.40
INT. 1 YEAR \$49.70 IS
49.7 ÷ 5.27 = 9.4%

CIRCUS: PRICE LOWERED BY 10% (x) ∴ 10% MORE TICKETS SOLD (y)
CALCULATE LOSS

$.9x \times 1.1y - xy = .01xy$

LOSS 1%

PERCENTAGE

DEPRECIATION 12%

OF VALUE. AFTER 5 YEARS

$ 4222

ORIGINAL VALUE

$4222 \div .88^5 =$ $ 8000

DEPRECIATION $8600

1ST YEAR 25% THEN 20%

FIND % LOSS AND VALUE AFTER 3 YEARS

$86 \times 75 = \cdots \times .8^2 =$

$ 4128 DIVIDE BY 86 =

48% LOSS 52%

COMPOUND INTEREST

- INTEREST ON INTEREST -

$ 1760 (1.) - 8.5% (2.) - 6 YEARS (3.)

WITHOUT FORMULA

LOOK AND SEE!

THE NUMBERS TELL THE STORY

$1760 \times 1.085^6 =$ $2871.38

PRINCIPAL

PRINCIPAL (1) + INTEREST AS A DECIMAL

RETAINER $ 185

+ .5% FIRST $1000 - THEN 1.5% ON $800

SALES $1800

INCOME

$ 185+5+12 = 202

LOAN $ 1500

@ 4.5% FLAT, 4 YEARS

PER MONTH

1500 + 15X4.5X4 = 1770

DIVIDE BY 48 ÷ $ 37.90

NOW INCOME $ 226

CALCULATE SALES $ x

COMMISSION $ 41

$5 ON FIRST $1000 THEN

IF 1.5% OF y = 36

$y = 36 \div .015$

$y = 2400 \therefore x =$ $ 3400

THESE PRACTICAL APPLICATIONS MUST ONLY BE DONE WHEN ROUTINE PROCEDURES ARE KNOWN !!

1. **POTATO SUMS** ANOTHER BARK RECIPE!

GIVING NAMES TO PROTOTYPES IS AN UNORTHODOX INNOVATION.

IT BREEDS FAMILIARITY AND CONFIDENCE.

ALL POTATO SUMS SOUND LIKE THIS:

IF 3kg COST $2.10,

5kg COST $x (A MYSTERY AMOUNT) (ALGEBRA) DO NOT USE ☐!

MENTAL EXERCISES WITH "ZIG-ZAG"

SET IT UP AS YOU THINK OR SAY IT:

IF 3 — IS — (THINK .70) 2.10

THEN 5 — IS → $x = \$3.50$

IT'S THE UNITARY METHOD IN ONE GO

AND EASIER WITH CALCULATOR SUMS!

2. POTATO SUMS — DONE MENTALLY

A. 141 BRICKS LAID IN 1 HOUR, HOW MANY IN 40 MINUTES

MENTALLY SIMPLIFIED:

THINK 47

$$\frac{3}{2} = \frac{141}{x} = 94$$

B. $\frac{3}{5}$ OF A NUMBER IS 9, FIND IT

$$\frac{\text{IF } 3 \text{ PARTS}}{5 \text{ PARTS}} \begin{matrix}\text{ARE}\\ = \\ \text{ARE}\end{matrix} \frac{9}{x} = 15$$

C. 108 km ON 9L, 360 ON xL

DIVIDE, THINK 12

$$\frac{108}{360 \div 12} = \frac{9}{x} = 30L$$

D. 3% OF A NUMBER IS 12. FIND IT.

$$\frac{3}{100} = \frac{12}{x} = 400$$

E. 2 HOURS \$10. 3 HOURS \$x

$$\frac{2}{3} = \frac{10}{x} = \$15$$

F. $\frac{2}{7}$ OF A NUMBER IS 6. FIND $\frac{1}{3}$.

$$\frac{2}{7} = \frac{6}{x} = 21 \therefore \frac{1}{3} \text{ OF } x = 7$$

3. POTATO SUMS BY CALCULATOR

TIMES, DIVIDE: CROSS-MULTIPLY SHORTCUT

A. 80 km PER HOUR. HOW FAR IN 32 MINUTES

DIVIDE

$$\frac{6}{32} = \frac{8}{x} \doteqdot 42.7 \text{ km}$$

TIMES APPR.

B.

EFFICIENCY! ONLY PRESS

$$\frac{3}{5} = \frac{16}{x} = 80 \div 3 \doteqdot 26.7$$

RECURRING: $26.\dot{6}$

C.

$$\frac{21}{46} = \frac{x}{111} \doteqdot 50.7$$

D. 29 L TO DO 376 km. HOW FAR ON 20 L.

WITHOUT WRITING IT DOWN!

$20 \times 376 \div 29 \doteqdot 259.3$ km

E. 3 FOR \$1.92. 5 FOR \$$x$ (A GIVEN TASK)

STUDENTS OFTEN PANIC WHEN ASKED QUESTIONS.

F. 182 km ON 20 L. x km ON 24 L

DIRECT $24 \times 182 \div 20 = 218.4$ km

RATE

1\. A PLANT GROWS FROM 3.5 cm TO 8.75 cm IN ONE WEEK.

DAILY GROWTH RATE

5.25 ÷ 7 = .75 cm

2\. POPULATION: FROM 10700 TO 12100 IN 5 YEARS

GROWTH RATE 1400 ÷ 5 = 280 PER ANNUM (P.A.) (LATIN)
PER YEAR

RELATE: ANNUAL, ANNIVERSARY.

3\. INSURANCE PREMIUM: $3.50 PER $100 P.A.

$10 000 WORTH WOULD COST $350 P.A.

4\. PETROL CONSUMPTION: LITRES PER 100 km

WHEN A CAR USES 95 L TO DO 1038 km,
CONSUMPTION IS x (MYSTERY AMOUNT)
NOTE: ALGEBRA USES PRONUMERALS, NOT □ !
(LETTERS FOR NUMERALS)

SET IT UP AS FOLLOWS & THINK:

IF 1038 IS 95
THEN 100 IS x ≑ 9.2 L
BY CALCULATOR

BARK RECIPE:

TIMES, DIVIDE

1. RATIO AND PROPORTION

1. SUGAR : FLOUR (TO) 4 : 20 = 1 : 5

THINK PARTS

USUALLY BY VOLUME

2. SIDES TRIANGLE 2 : 3 : 4 THINK 9 PARTS

PERI METER 81 cm

GREEK: AROUND MEASURE

SIDES ARE (81 ÷ 9 X 2), (81 ÷ 9 X 3) & (81 ÷ 9 X 4) cm

CHECK: 18 + 27 + 36 = 81 ✓

NOTE: PERI SCOPE
AROUND LOOK

3. SHADOW SUM: TREE 6 m, SHADOW 2 m
BUILDING H m, SHADOW 7 m

SET IT UP AS FOLLOWS & THINK.

IF 2 — IS → 6

THEN 7 — IS → H = 21 m (MENTALLY)

4. 2 L (2000 ml) JUICE A : B : C = 2 : 3 : 5

APPLE 2000 ÷ 10 X 2 = 400 ml

BANANA 600 ml

CARROT + 1000 ml (1 L)

CHECK 2000 ✓

2. RATIO AND PROPORTION

1. $$27 : 12 = 9 : 4$$

SIMPLIFIED

2. $5, 8, x, 32$ ARE IN PROPORTION

$$8x = 160 \quad \therefore \quad x = 20$$

THEREFORE

3. DIVIDE \$56 INTO RATIO 3:5

THINK 8 PARTS

3 PARTS $= 56 \div 8 \times 3 = \$21$

5 PARTS $= 56 \div 8 \times 5 = \$35$

CHECK + $\overline{56}$ ✓

4. CAR 4.2 m LONG, 1.5 m HIGH, MODEL 70 mm LONG, H HIGH

$$\frac{420 \text{ (cm)}}{7} = 60 = \frac{150 \text{ (cm)}}{H} \therefore 60H = 150$$

$$H = 2.5 \text{ cm} = 25 \text{ mm}$$

ROMAN NUMERALS

7 INGREDIENTS

HOLY NUMBER

I	V	X	L
ONE	HALF THE TEN FIVE	DIX = 10 TEN	L FOR LIRE THINK V FOR 5 TURNED 45° 5X10 = 50
	$\bar{V}$ = 5000	$\bar{X}$ = 10 000	$\bar{L}$ = 50 000

C	D	M
ALWAYS LOOK FOR 'CLUES' TO REMEMBER	DAFTERC	MILE: 1000 ROMAN SOLDIER STEPS
CENT 100	DEMI 500 HEMI SEMI HALF	MILLE 1000
$\bar{C}$ = 100 000	$\bar{D}$ = 500 000	$\bar{M}$ = 1000 000

NEVER USE 1 SYMBOL MORE THAN 3 TIMES IN A ROW
ALTHOUGH IT IS DONE (CATHEDRALS, ETC.)

SMALL BIG −
BIG SMALL +

I	II	III	IV	V
VI	VII	VIII	IX	X
X	XX	XXX	XL	L
LX	LXX	LXXX	XC	C
CC	CCC	CD	D	DC

DCC	DCCC	CM	M	MC
MM	CMXCIX		999	

ANNO DOMINI (A.D.)
IN THE YEAR OF OUR LORD

MCMXCII

1 9 9 2

A.D. NOT AFTER DEATH

B.C. IS BEFORE CHRIST
OR B.C.E.

SCALE

1. CENTIMETRE (cm) = $\frac{1}{100}$ METRE (m)

IF 1 cm REPRESENTS 1 m (100 cm),

SCALE = 1 : 100

ONE TO A HUNDRED

2. MILLIMETRE (mm) = $\frac{1}{1000}$ METRE

METRIC ↑

IF 1 mm REPRESENTS 2 m (2000 mm),

SCALE = 1 : 2000

3. DIAMETER MOON 3500 km, EARTH 12 800 km

SCALE 1 : 100 000 000

MEANS 1 cm REPRESENTS 1000 km

A CIRCLE, D = 3.5 cm REPRESENTS THE MOON.

A CIRCLE, D = 12.8 cm REPRESENTS THE EARTH.

4. SCALE 1 : 10 000 000 ∴ 7 cm ⟶ 700 km

IF 3 cm REPRESENTS 12 m,

4 cm REPRESENTS 16 m.

9	
1	8
2	7
3	6
5	4

10	
ONE	NINE
TWO	EIGHT
THREE	SEVEN
FOUR NEXT	SIX EVEN
5 2 HANDS	5

X+1	
3+1	4
2+1	3
5+1	6
4+1	5
7+1	8
6+1	7
10+1	11

2+EVEN	
2+6	8
2+10	12
2+4	6

2+ODD	
2+5	7
2+9	11
2+3	5

DOUBLES

4+4=2X4

10+10	20
3+3	6
1+1	2
4+4	8
7+7	14
2+2	4
5+5	10
8+8	16
6+6	12

DOUBLES+1	
3+4	7
6+7	13
5+6	11
7+8	15

X+9	
3+9	12
5+9	14
4+9	13

SORTING	
6+9	15
8+9	17
7+9	16
9+9	18

X+TEEN	
3+10	13
5+10	15
7+10	17
9+10	19
8+10	18
4+10	14
4+7	11
3+8	11
5+7	12
4+8	12
8+6	14
3+5 5+8	8 13

SORTING				12-6	6	13-9	4	13-5	8
x - 1		CONSECUTIVE ODD		10-5	5	15-9	6	11-6	4+1
8-1	7	7-5	2	9 PARTNER		14-9	5	13-7	6
2-1	1	5-3	2	9-8	1	DIGITS - THEIR SUM		14-6	8
7-1	6	EVEN-2		9-6	3	18-9	9	13-6	7
3-1	2	8-2	6	9-4	5	17-8	9	11-7	4
6-1	5	4-2	2	9-2	7	16-7	9	12-7	3+2
4-1	3	6-2	4	9-5	4	15-6	9	15-7	8
5-1	4	ODD-2		9-1	8	14-5	9	11-8	2+1
DIFFERENCE 2 CONSECUTIVES		7-2	5	9-7	2	13-4	9	15-8	7
8-7	1	3-2	1	9-3	6	12-3	9	12-8	4
4-3	1	5-2	3	ADD THE DIGITS		11-2	9	14-8	6
7-6	1	HALVES		18-9	9	11-3	7+1	13-8	5
5-4	1	16-8	8	11-9	2	11-4	6+1	7-3	4
CONSECUTIVE EVEN		6-3	3	17-9	8	12-4	6+2	8-5	3
8-6	2	14-7	7	12-9	3	11-5	5+1	7-4	3
6-4	2	8-4	4	16-9	7	12-5	5+2	8-3	5

TASK BASED RECOGNITION EXERCISES

10X1	10	9X2	18	2X2	4	10X6	60
9X4	36	2X3	6	10X7	70	2X4	8
6X2	12	10X3	30	9X3	27	3X4	12
4X7	28	6X4	24	8X4	32	10X2	20
5X2	10	9X5	45	6X6	36	7X7	49
7X3	21	5X5	25	9X7	63	6X8	48
10X4	40	2X7	14	12X11	121	3X3	9
8X3	24	10X5	50	5X6	30	9X6	54
5X4	20	4X4	16	9X9	81	10X8	80
10X9	90	9X8	72	10X10	100	5X8	40
3X5	15	3X6	18	2X8	16	5X7	35
7X6	42	7X8	56	8X8	64	23X100	2300

Numbers In A Nutshell
With
Aart Bark

Author's Background

Born 20.12.1928

1. **H.S.C. (HOLLAND)** 1947
   4 Unit Maths, Mechanics, Technical Drawing
   Physics, Chemistry, Biology, Political Economy
   History, Geography, Dutch, English, French
   German, Art, P.E.
2. Certificate of Ability, Nautical College Holland, 1949
3. Diploma 3rd. Mate, Sea Going Trade Holland 1951
4. Diploma 2nd. Mate, Sea Going Trade Holland 1954
5. Spanish Commercial Correspondence Holland 1954
6. French Commercial Correspondence Holland 1958
7. English Commercial Correspondence Holland 1961
8. Language Studies: Friesian, Italian, B.A. French
9. High School Teacher: English & French Holland 2 yrs
10. High School Teacher Australia 14 yrs
    De La Salle, Ashfield
    Latin, French & English.
    St. Dominic's, Kingswood
    Creative Writing, English, Subject Master Technical Drawing
    Patrician Brothers, Granville
    Creative Writing, Mathematics, Subject Master Music
    Oakhill College, Castle Hill
    Creative Writing, Mathematics, Subject Master Technical Drawing, French & Art
11. Insurance & Real Estate Agent (Finance)
12. Owner Builder (Rammed Earth)
13. Hawkesbury Adult Education Creative Writing, Spanish.
14. Professional Musician Accordion, Flamenco Guitar.
15. Author of Textbooks English & Mathematics
16. Private Tutor since 1976: K-12
17. Soccer Coach

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Numbers in a Nutshell

- This book has been my private program, fine-tuned over a period of 40 years.
- Its success is the result of Creative Simplicity.
- This unique Numeracy Course is the only one that shows students **How to Remember** mathematical facts and procedures.
- By pinpointing umpteen idiosyncracies of students from different schools, I have discovered a simple strategy to diffuse the "I don't understand" ones. It consists of replacing Programmed Negative Thought by Positive Vocal Repetition!
- As an ex-officer in the Dutch Merchant Navy, I realised that, in a modern "Instant Satisfaction" society, **the performance of students can only improve** by using a short practical approach rather than a lengthy academic one.
- Instead of solely depending on **unreliable** intelligence, I teach students to **See** and **Do** without any assumptions!
- By only working with **Proto Types**, the entire prescribed material for Primary Schools is contained in only 130 pages!
- It means that Repetition – an important necessity in Proffessional Memory Training – can now occur at more frequent intervals. **Less haste, more speed**.
- However, the greatest advantage of this course lies in the fact that teachers will have enough time to **consolidate each topic** before attempting the next one.

A. Bark
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